NSTX Upgrade Flexible Strap Assy:
Single Lamination EMAG Stress Analysis

1.0 Problem Description

To determine if the baseline NSTX flexible strap design is adequate to meet the NSTX Structural Design critieria, specifically, the fatique requirements of section I-4.2 for 3000 full power and 30,000 two-thirds full-power pulses without failure.

2.0 Given

The baseline laminated flexible strap assy design is shown in Figure 1. The outer lamination is assumed worst-case and will be analyz below.

Number of laminations - $n := 38$

Total Current - $I := 130000 \cdot A$

Current per lamination - $I_{lam} := \frac{I}{n} = 3.421 \times 10^{3} \cdot A$

Poloidal field flux density - $B_{pol} := .3 \cdot T$

Toroidal field flux density - $B_{tor} := 1 \cdot T$

Thermal displacements - $\delta_{vert} := .3 \cdot in$ $\quad \delta_{hor} := .018 \cdot in$

Outside radius - $R_o := 5.688 \cdot in$

Width - $h := 2 \cdot in$

Thickness - $d := .06 \cdot in$

Radius of curvature - $R := R_o - \frac{d}{2}$ $\quad R = 5.658 \cdot in$

Material: Copper

Elastic Modulus - $E := 17 \cdot 10^{6} \cdot psi$

Poisson's ratio - $\nu := .3$

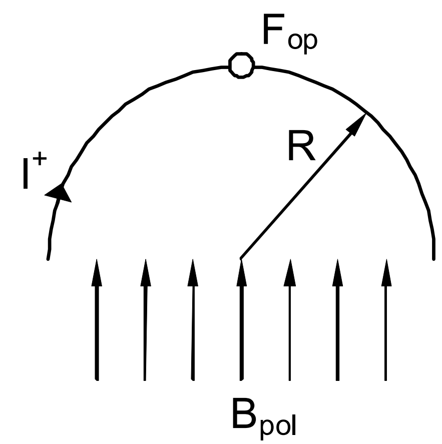
Modulus of rigidity - $G := \frac{E}{2\cdot(1+\nu)} = 6.538\times 10^{6}\,psi$

3.0 Calculated EMAG Loads

3.1 Out-of-Plane Force Load (z-direction) - F_{op}

$F_{op} := 2\cdot I_{lam}\cdot B_{pol}\cdot R = 295\cdot N$ [1]

$F_{op} = 66.3\,lbf$



3.2 In-Plane Pressure Load (y-direction) - p_{ip}

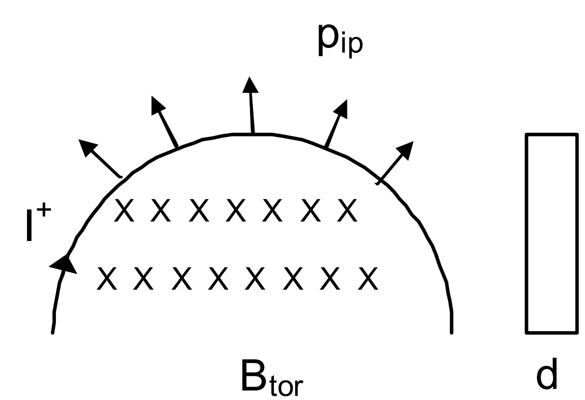
$F_{ip\,/L} := I_{lam}\cdot B_{tor}$ [1]

$F_{ip\,/L} = 3.421\times 10^{3}\cdot\frac{N}{m}$

$F_{ip\,/L} = 19.5\cdot\frac{lbf}{in}$

$p_{ip} := \frac{\left(F_{ip\,/L}\right)}{h}$

$p_{ip} = 9.8\,psi$



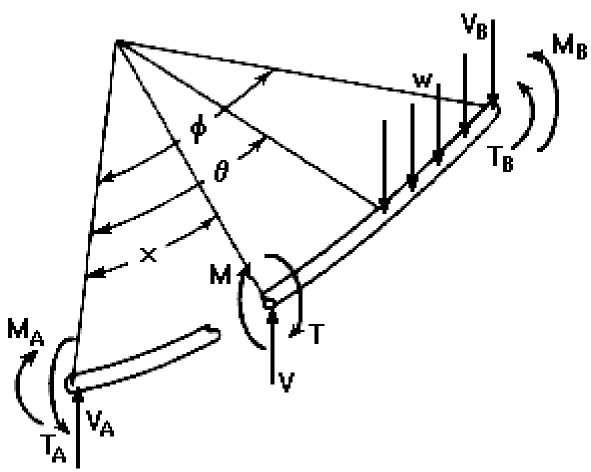
4.0 Stress Analysis

4.1 Out-of-Plane Emag Load Stresses

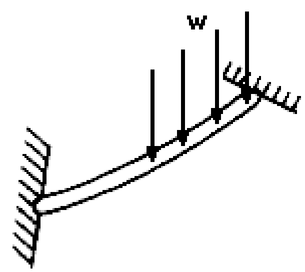
The out-of-plane load results in out-of-plane bending, torsion, and direct shear.

This following is from ***Roark's Formulas for Stress and Strain, 7th Edition,*** Table 9.4, Formulas for Curved Beams of Compact Cross Section Loaded Normal to the Plane of Curvature, Case 4e: Uniformly distributed lateral load, both ends fixed.

Uniformly distributed lateral load



- **Case 4e Right end fixed, left end fixed**



Angle from left end to start of loading - $\theta := 0 \cdot rad$

Angle subtended by span of the beam - $\phi := \pi \cdot rad$

Length of beam - $L := R \cdot \phi = 17.775 \cdot in$

Distributed load (force/length) - $w := \frac{F_{op}}{L} = 3.731 \cdot \frac{lbf}{in}$

Area moment of inertia about bending axis - $I := \frac{d \cdot h^3}{12} = 0.04 \cdot in^4$

Torsional stiffness (rectanglular section) - $K := a \cdot b^3 \cdot \left[\frac{16}{3} - 3.36 \cdot \frac{b}{a} \cdot \left(1 - \frac{b^4}{12 \cdot a^4}\right)\right]$ (a >> b)

where: $a := \frac{h}{2}$ $b := \frac{d}{2}$

$K = 1.413 \times 10^{-4} \cdot in^4$

Solid rectangular section

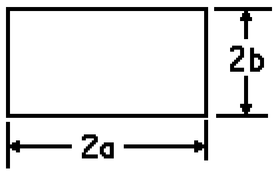


Table 9.4 Equation constants:

$$\beta := \frac{E \cdot I}{G \cdot K} = 736.1$$

$$C_1 := \frac{1+\beta}{2} \cdot \phi \cdot \sin(\phi) - \beta \cdot (1 - \cos(\phi)) = -1.472 \times 10^3$$

$$C_2 := \frac{1+\beta}{2} \cdot (\phi \cdot \cos(\phi) - \sin(\phi)) = -1.158 \times 10^3$$

$$C_3 := -\beta \cdot (\phi - \sin(\phi)) - \frac{1+\beta}{2} \cdot (\phi \cdot \cos(\phi) - \sin(\phi)) = -1.155 \times 10^3$$

$$C_4 := \frac{1+\beta}{2} \cdot \phi \cdot \cos(\phi) + \frac{1-\beta}{2} \cdot \sin(\phi) = -1.158 \times 10^3$$

$$C_5 := -\frac{1+\beta}{2}\cdot\phi\cdot\sin(\phi) = -1.418\times10^{-13}$$

$$C_6 := C_1$$

$$C_7 := C_5$$

$$C_8 := \frac{1-\beta}{2}\cdot\sin(\phi) - \frac{1+\beta}{2}\cdot\phi\cdot\cos(\phi) = 1.158\times10^{3}$$

$$C_9 := C_2$$

$$C_{a2} := \frac{1+\beta}{2}\cdot[(\phi-\theta)\cdot\cos(\phi-\theta) - \sin(\phi-\theta)] = -1.158\times10^{3}$$

$$C_{a3} := -\beta\cdot(\phi-\theta-\sin(\phi-\theta)) - C_{a2} = -1.155\times10^{3}$$

$$C_{a12} := \frac{1+\beta}{2}\cdot[(\phi-\theta)\cdot\sin(\phi-\theta) - 2 + 2\cdot\cos(\phi-\theta)] = -1.474\times10^{3}$$

$$C_{a13} := \beta\cdot\left[1-\cos(\phi-\theta) - \frac{(\phi-\theta)^2}{2}\right] - C_{a12} = -686.1$$

$$C_{a16} := C_{a3}$$

$$C_{a19} := C_{a12}$$

4.1.1 Direct Shear Stress - τ_{op_direct}

Direct shear force at end A - $V_A := w\cdot R\cdot\frac{C_{a13}\cdot(C_4\cdot C_8 - C_5\cdot C_7) + C_{a16}\cdot(C_2\cdot C_7 - C_1\cdot C_8) + C_{a19}\cdot(C_1\cdot C_5 - C_2\cdot C_4)}{C_1\cdot(C_5\cdot C_9 - C_6\cdot C_8) + C_4\cdot(C_3\cdot C_8 - C_2\cdot C_9) + C_7\cdot(C_2\cdot C_6 - C_3\cdot C_5)}$

$$V_A = 33.2\,\text{lbf}$$

Direct shear force at end B - $V_B := V_A - w \cdot R \cdot (\phi - \theta) = -33.158\,\text{lbf}$

$$\tau_{op\_direct} := \frac{V_A}{d \cdot h} = 276.3\,\text{psi}$$

4.1.2 Bending Stress - σ_{op_bend}

Bending moment at end A - $M_A := w \cdot R^2 \cdot \frac{C_{a13} \cdot (C_5 \cdot C_9 - C_6 \cdot C_8) + C_{a16} \cdot (C_3 \cdot C_8 - C_2 \cdot C_9) + C_{a19} \cdot (C_2 \cdot C_6 - C_3 \cdot C_5)}{C_1 \cdot (C_5 \cdot C_9 - C_6 \cdot C_8) + C_4 \cdot (C_3 \cdot C_8 - C_2 \cdot C_9) + C_7 \cdot (C_2 \cdot C_6 - C_3 \cdot C_5)}$

$M_A = -119.4 \cdot \text{in} \cdot \text{lbf}$

Bending moment at end B - $M_B := -M_A = 119.4 \cdot \text{in} \cdot \text{lbf}$

$$c := \frac{h}{2} = 1 \cdot \text{in}$$

$$\sigma_{op\_bend} := \frac{M_A \cdot c}{I} = -2.986 \times 10^3\,\text{psi}$$

4.1.3 Torsional Stress - τ_{op_tor}

Twisting moment at end A - $T_A := w \cdot R^2 \cdot \frac{C_{a13} \cdot (C_6 \cdot C_7 - C_4 \cdot C_9) + C_{a16} \cdot (C_1 \cdot C_9 - C_3 \cdot C_7) + C_{a19} \cdot (C_3 \cdot C_4 - C_1 \cdot C_6)}{C_1 \cdot (C_5 \cdot C_9 - C_6 \cdot C_8) + C_4 \cdot (C_3 \cdot C_8 - C_2 \cdot C_9) + C_7 \cdot (C_2 \cdot C_6 - C_3 \cdot C_5)}$

$T_A = 35.5 \cdot in \cdot lbf$

Twisting moment at end B - $T_B := -T_A = -35.5 \cdot in \cdot lbf$

$$\tau_{op\_tor} := \frac{3 \cdot T_A}{8 \cdot a \cdot b^2} \cdot \left[1 + 0.6095 \cdot \frac{b}{a} + 0.8865 \cdot \left(\frac{b}{a}\right)^2 - 1.8023 \cdot \left(\frac{b}{a}\right)^3 + 0.9100 \cdot \left(\frac{b}{a}\right)^4 \right]$$

$\tau_{op\_tor} = 1.509 \times 10^4 \, psi$

4.1.4 Combined Equivalent von Mises Stress - σ_{e_op}

$$\sigma_{e\_op} := \left[\frac{1}{2} \cdot \left[(2 \cdot \sigma_{op\_bend})^2 + 6 \cdot (\tau_{op\_tor} + \tau_{op\_direct})^2 \right] \right]^{.5} = 2.695 \times 10^4 \, psi$$

4.1.5 ANSYS FEA Results

The results of an ANSYS FEA model of a single lamination with the out-of-plane Emag distributed force load only applied to the concave surface of the lamination and with fixed constraints on the straight legs is shown in Figure 2:

$\sigma_{e\_op\_ANSYS} = 2.917 \times 10^4$ psi

4.2 In-Plane Emag Load Stresses

The in-plane Emag pressure load produces an in-plane tangential hoop stress, and an out-of-plane bending stress due to the offset/ joggle in the lamination.

4.2.1. Hoop stress - σ_{ip_hoop}

$$\sigma_{ip\_hoop} := \frac{p_{ip} \cdot R}{d} = 921.1\,\text{psi}$$

4.2.2 Offset Bending Stress - $\sigma_{ip\_offset\_bend}$

Assume offset/ joggle can be modeled as a short straight beam with a uniform distributed over the length and fixed at both ends.

Offset length $l_{offset} := .94 \cdot in$ $\quad F_{ip\,/L} = 19.535 \cdot \frac{lbf}{in}$

In-plane Emag force - $F_{ip} := \left(F_{ip\,/L}\right) \cdot L = 347.2\,lbf$

Effective offset distributed load - $w_{offset} := \frac{F_{ip}}{l_{offset}} = 369.4 \cdot \frac{lbf}{in}$

Bending moment $M_{offset} := \frac{w_{offset} \cdot l_{offset}^{2}}{12} = 27.2 \cdot in \cdot lbf$

$$\sigma_{ip\_offset\_bend} := \frac{M_{offset} \cdot c}{I} = 680\,\text{psi}$$

4.2.3 Combined Equivalent von Mises Stress - σ_{e_ip}

$$\sigma_{e\_ip} := \left|\sigma_{ip\_hoop}\right| + \left|\sigma_{ip\_offset\_bend}\right| = 1.601 \times 10^{3}\,\text{psi}$$

4.2.4 ANSYS FEA Results

The results of an ANSYS FEA model of a single lamination with the in-plane Emag pressure load only applied to the concave surface of the lamination and with fixed constraints on the straight legs is shown in Figure 3:

$\sigma_{e\_ip\_ANSYS} = 5.989 \times 10^3$ psi

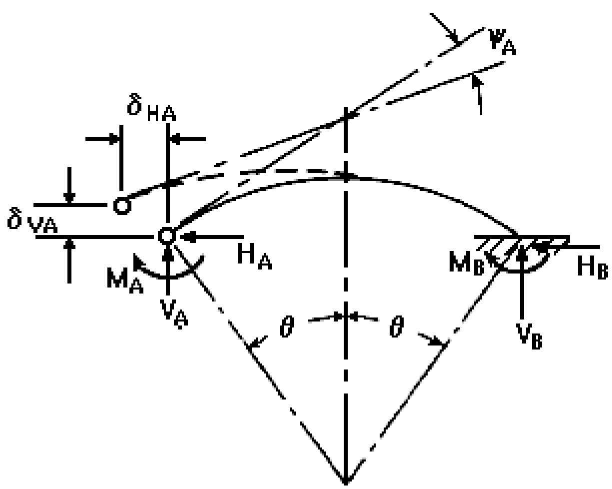
4.3 Thermal Displacement Stresses

The horizontal displacement is small and asumed to be negligible

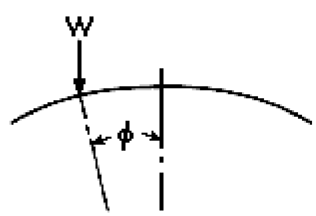
The vertical thermal displacement results in in-plane bending and simple tension.

4.3.1 Vertical Thermal Displacement Bending Stress - σ_{therm_bend}

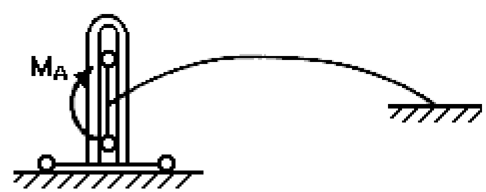
The following is from ***Roark's Formulas for Stress and Strain, 7th Edition,*** Table 9.3, Reaction and Deformation Formulas for Circular Arches, Cases 5a and 11: Concentrated vertical loading, left end restrained against rotation only, right end fixed..



5a. Concentrated vertical load



11. Left end restrained against rotation only, right end fixed



Area properties:

From Table 9.1, Formulas for curved beams subject to bending in the plane of curvature, Ref. No. 1, Solid rectangular section

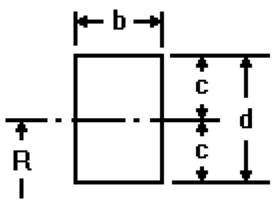
Radius of curvature - $R = 5.658 \cdot in$

Height of rectangular section - $d = 0.06 \cdot in$

Width of rectangular section - $b := h \qquad b = 2\,in$

$\frac{R}{d} = 94.3$ R/d > 8, consider as thin beam

Solid rectangular section



Half height - $c := \frac{d}{2} = 0.03 \cdot in$

Moment of inertia about centriodal axis perpindicular to plane of curvature - $I_c := \frac{b \cdot d^3}{12} = 3.6 \times 10^{-5} \cdot in^4$

Area - $A := b \cdot d = 0.12 \cdot in^2$

Distance from centroidal axis to neutral axis measured toward the center of curvature - $h := \frac{I_c}{R \cdot A} = 5.302 \times 10^{-5} \cdot in$

Ratio of actual stress in extreme fiber on concave side to ficticious - $k_i := \frac{\sigma_i}{\sigma}$

stress calculated for straight beam

$$k_i := \left(\frac{1}{\frac{3 \cdot h}{c}}\right) \cdot \left(\frac{1 - \frac{h}{c}}{\frac{R}{c} - 1}\right) = 1.004$$

Ratio of actual stress in extreme fiber on convex side to ficticious stress calculated for straight beam -

$$k_o := \frac{\sigma_o}{\sigma}$$

$$k_o := \left(\frac{1}{\frac{3 \cdot h}{c}}\right) \cdot \left(\frac{1 + \frac{h}{c}}{\frac{R}{c} + 1}\right) = 0.996$$

Shape factor for rectagular section - $F := \frac{6}{5} = 1.2$

Table 9.3 Equation constants:

$$\alpha := \frac{I_c}{A \cdot R^2} = 9.371 \times 10^{-6}$$

$$\beta := \frac{F \cdot E \cdot I_c}{G \cdot A \cdot R^2} = 2.924 \times 10^{-5}$$

$$k_1 := 1 - \alpha + \beta = 1$$

$$k_2 := 1 - \alpha = 1$$

Half angle subtended by arch - $\theta := \frac{\pi}{2} \cdot rad = 1.571 \cdot rad$

$s := \sin(\theta) = 1$

$c := \cos(\theta) \qquad c = 0$

Angle measured counterclockwise from the midspan of the arch to the start of the load - $\phi := \frac{\pi}{2} \cdot rad$

$n := \sin(\phi) \qquad n = 1$

$e := \cos(\phi) = 0$

$B_{HH} := 2 \cdot \theta \cdot c^2 + k_1 \cdot (\theta - s \cdot c) - k_2 \cdot 2 \cdot s \cdot c = 1.571$

$B_{HV} := -2 \cdot \theta \cdot s \cdot c + k_2 \cdot 2 \cdot s^2 = 2$

$B_{VH} := B_{HV} = 2$

$B_{HM} := -2 \cdot \theta \cdot c + k_2 \cdot 2 \cdot s = 2$

$B_{MH} := B_{HM} = 2$

$B_{VV} := 2 \cdot \theta \cdot s^2 + k_1 \cdot (\theta + s \cdot c) - k_2 \cdot 2 \cdot s \cdot c = 4.712$

$B_{VM} := 2 \cdot \theta \cdot s = 3.142$

$B_{MV} := B_{VM} = 3.142$

$$B_{MM} := 2\cdot\theta = 3.142$$

Initial guess of concentrated load - $W := -.645\cdot lbf$

Loading terms:

$$LF_H := W\cdot\left[-(\theta+\phi)\cdot c\cdot n + \frac{k_1}{2}\cdot\left(c^2 - e^2\right) + k_2\cdot(1 + s\cdot n - c\cdot e)\right] = -1.29\,lbf$$

$$LF_V := W\cdot\left[(\theta+\phi)\cdot s\cdot n + \frac{k_1}{2}\cdot(\theta+\phi+s\cdot c+n\cdot e) - k_2\cdot(2\cdot s\cdot c - s\cdot e + c\cdot n)\right] = -3.04\,lbf$$

$$LF_M := W\cdot\left[(\theta+\phi)\cdot n + k_2\cdot(e-c)\right] = -2.026\,lbf$$

Formulas for horizontal and vertical deflections, reaction moment, horizontal and vertical end reactions and angular rotation at the left edge

Angular rotation: $\psi_A := 0\cdot in$

Horizontal reaction: $H_A := 0\cdot lbf$

Vertical reaction: $V_A := 0\cdot lbf$

Because the above equal zero:

Reaction moment: $M_A := R\cdot\frac{LF_M}{B_{MM}} = -3.6\cdot in\cdot lbf$

Horizontal deflection: $\delta_{HA} := \frac{R^3}{E\cdot I_c}\cdot\left(B_{HM}\cdot\frac{M_A}{R} - LF_H\right) = 0\cdot in$

Vertical deflection:

$$\delta_{VA} := \frac{R^3}{E \cdot I_c} \cdot \left(B_{VM} \cdot \frac{M_A}{R} - LF_V \right) = 0.3 \cdot in$$

If $\delta_{VA} = \delta_{vert}$, initial W guess is correct

$$M_B := -M_A + 2 \cdot R \cdot W = -3.649 \cdot in \cdot lbf$$

$$\sigma_{therm\_bend} := \frac{M_B \cdot \frac{d}{2}}{I_c} = -3.041 \times 10^3 \, psi$$

4.3.2 Vertical Thermal Displacement Tension Stress - $\sigma_{therm\_tension}$

$$\sigma_{therm\_tension} := \frac{-W}{b \cdot d} = 5.375 \, psi$$

4.3.4 Combined Equivalent von Mises Stress - σ_{e_therm}

$$\sigma_{e\_therm} := \left| \sigma_{therm\_bend} \right| + \left| \sigma_{therm\_tension} \right| = 3.047 \times 10^3 \, psi$$

4.3.5 ANSYS FEA Results

The results of an ANSYS FEA model of a single lamination with the thermal displacement loads only and with vertically guided constraight on the left end and fixed constraint on the right end is shown in Figure 4:

$$\sigma_{e\_therm\_ANSYS} = 3.980 \times 10^3 \text{ psi}$$

4.4 Combined Emag and Thermal Displacement Stress

4.4.1 Calculated Combined von Mises Stress

$$\sigma_{e\_tot} := \sigma_{e\_op} + \sigma_{e\_ip} + \sigma_{e\_therm} = 3.16 \times 10^{4}\ psi$$

4.4.2 ANSYS FEA Results

The results of an ANSYS FEA model of a single lamination with the combined Emag and thermal displacement loads with a vertically guided constraint on the left end and a fixed constraint on the right end is shown in Figure 5:

$$\sigma_{e\_tot\_ANSYS} = 3.551 \times 10^{4}\ psi$$

$$K := a \cdot b^3 \cdot \left[\frac{16}{3} - 3.36 \cdot \frac{b}{a} \cdot \left(1 - \frac{b^4}{12 \cdot a^4} \right) \right]$$