PPPL Calculation Form - No: NSTXU-CALC-133-25

Calculation # NSTXU-CALC-133-25 Revision # 0 ____ WP #, if any (ENG-032) 2254 ____

Purpose of Calculation: (Define why the calculation is being performed.)

The purpose of this calculation is to document the development of interim insulation properties and winding pack "Smeared" properties appropriate for stress analysis of both the coils the coil support structures.

Codes and versions: (List all codes, if any, used)

ANSYS 18.2

References (List any source of design information including computer program titles and revision levels.)

[1] NSTXU-RQMT-RD-012-00, Inner PF Coil Interfaces to Coil Support Designs and Cooling Systems, S. Gerhardt, 2017.
[2] "Effective mechanical properties of EM Composite conductors: and analytical and finite element modeling approach", W. Sun, J. T. Tzeng, Composite Structures 58 (2002) 411-421.
[3] NSTXU-CALC-133-26-00, OH/PF and TF Insulation Mechanical Properties and Allowables, March 2018.
[4] MEMO-SEI-180223PHT01, "OH Insulation Tests Applicability to Inner PF Qualification", P. Titus, 2018.

Assumptions (Identify all assumptions made as part of this calculation.)

The insulation properties are derived from a "first principals" treatment of the epoxy/glass/Kapton system, and analytic derivations of the winding pack composite systems. These have been shown to be reasonably close to NSTXU CTD 425 measured coil properties for the OH in service, OH qualification tests, PF-1aL and sections of PF-1aU. Recent test results from small samples by CTD and tests on the PPPL "log" sample have not been incorporated into the analysis models. While similar behaviors are evident in the calculated and measured properties, it is assumed that the small differences in analysis properties and final measured properties will not alter the conclusions of the coil and polar region analyses.

Calculation (Calculation is either documented here or attached)

Please see attached main body of this document. The calculations are based on reference [2] and implemented in MATHCAD. The calculation set-up and results follow.

Conclusion (Specify whether or not the purpose of the calculation was accomplished.)

Specified properties in the attachment to the requirements document are adequate for initial analyses and will be subject to verification by including all of the latest CTD data.

Cognizant Individual (or designee) printed name, signature, and date Steve Raftopoulos Digitally signed by Steve Raftopoulos Date: 2018.09.12 09:54:24 -04'00'

Preparer's printed name, signature and date Yuhu Zhai Digitally signed by Yuhu Zhai Date: 2018.09.11 15:20:38 -04'00'

I have reviewed this calculation and, to my professional satisfaction, it is properly performed and correct.

Checker's printed name, signature, and date

Peter H. Titus

Digitally signed by Peter H. Titus
Date: 2018.09.12 16:45:58 -04'00'

National Spherical Torus eXperiment - Upgrade

NSTX-U

Calculation of Insulation and Coil Winding Pack Orthotropic for Initial Inner PF Coil and Polar Region Analyses

NSTXU-CALC-133-25 -00
March 28 2018

Thomas Willard — Digitally signed by Thomas Willard Date: 2018.03.29 11:16:03 -04'00'

Yuhu Zhai — Digitally signed by Yuhu Zhai Date: 2018.03.29 11:31:22 -04'00'

Prepared By
Tom Willard, Yuhu Zhai

Peter H. Titus — Digitally signed by Peter H. Titus Date: 2018.03.29 12:52:09 -04'00'

Reviewed By
Peter Titus

Steve Raftopoulos — Digitally signed by Steve Raftopoulos Date: 2018.03.29 13:44:03 -04'00'

Approved By – Responsible Engineer
Steve Raftopoulos

NSTX-U CALCULATION

Record of Changes

Rev.	Date	Description of Changes	Revised by
0	3/28/18	Initial Release	

NSTX-U Calculation Form

Purpose of Calculation:

The purpose of this calculation is to document the development of Interim insulation properties and winding pack “Smeared” properties appropriate for stress analysis of both the coils and

References:

[1] "Inner PF Coil Interfaces to Coil Support Designs and Cooling Systems" NSTXU RQMT-RD-012-00, Stefan Gerhardt
[2] "Effective mechanical properties of EM Composite conductors: and analytical and finite element modeling approach" W. Sun, J.T. Tzeng Composite Structures 58 (2002) 411-421
[3] OH/PF and TF Insulation Mechanical Properties and Allowables NSTXU-CALC-133-26-00, March 2018
[4] “OH INSULATION TESTS APPLICABILITY TO INNER PF QUALIFICATION” MEMO SEI-180223PHT01

Assumptions:
The insulation properties are derived from a “first principals” treatment of the epoxy/glass/Kapton system, and analytic derivations of the winding pack composite systems. These have been shown to be reasonably close to NSTXU CTD 425 measured coil properties for the OH in service, OH qualification tests, PF1aL and sections of PF1aU. Recent results from small samples by CTD and tests on the PPPL “log” test have not been incorporated into the analysis models, and while similar behaviors are evident in the calculated and measured properties, it is assumed that the small differences in analysis properties and final measured properties will not alter the conclusions of the coil and polar region analyses

Calculation:
The calculations are based on reference [2] and implemented in MATHCAD. The calculation set-up and results follow.

Conclusion:

Specified properties in the attachment to the requirements document are adequate for initial analyses and will be subject to verification by including all of the latest CTD data.

From Reference [1]:

1 Appendix 1: Coil Mechanical Properties

b. Coil insulation composite mechanical properties used for deriving the coil pack smeared mechanical properties for the coil support analysis are provided in Tables A1-2 through A1-3. *Note that these parameters may evolve as additional test data becomes available*.

Table A1-2*: Coil insulation composite coefficient of thermal expansion : X-conductor tangent; Y - thru thickness; and Z – conductor axis (toroidal)*

Quantity	Units	Value
CTE in normal direction, $\alpha_{\mathbf{x}}$	C^{-1}	25e-6
CTE in fill direction, $\alpha_{\mathbf{y}}$	C^{-1}	10e-6
CTE in wrap direction, $\alpha_{\mathbf{z}}$	C^{-1}	10e-6

Table A1-3*: Coil insulation composite elastic modulus: X-conductor tangent; Y – thru thickness; and Z-conductor axis (toroidal direction)*

Quantity	Units	Value
Ex	GPa	8.01
Ey	GPa	4.49
Ez	GPa	14.71
Gxy	GPa	1.88
Gyz	GPa	1.88
Gxz	GPa	3.88
$\mathbf{v}_{\mathbf{xy}}$		0.31
$\mathbf{v}_{\mathbf{yz}}$		0.31
$\mathbf{v}_{\mathbf{zx}}$		0.42

c. Smeared coil mechanical properties to be used for support analysis are provided in Tables A1-4 through A1-6. For implementing tables A1-4 through 6 refer to Figure A1-1 for the unit cell cylindrical coordinate system orientation. *Note that these parameters may evolve as additional test data becomes available*.

Fig A1-1*: Unit cell cylindrical coordinate system*

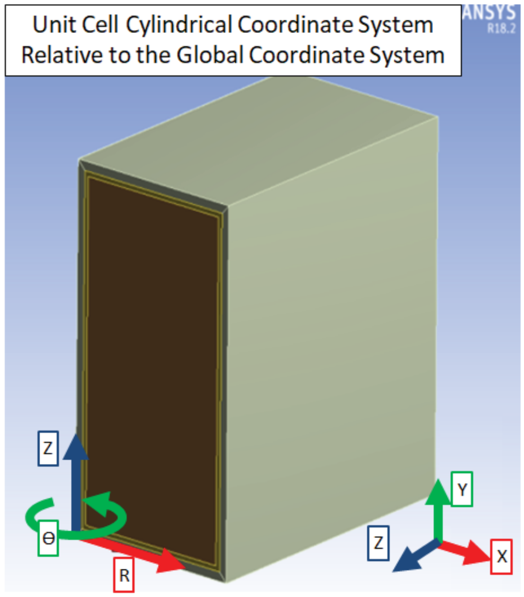


Table A1-4*: "Smeared" coil pack coefficient of thermal expansion*

Quantity	**Units**	**Value**
CTE in radial direction, max	C-1	18.8e-06
CTE in radial direction, min	C^{-1}	18.8e-06
CTE in toroidal direction, max	C^{-1}	17.8-06
CTE in toroidal direction,	C^{-1}	17.8-06

min		
CTE in vertical direction, min	C^{-1}	18.3-06
CTE in vertical direction, max	C^{-1}	18.3e-06

***Table A1-5**: "Smeared" coil pack modulus values*

Quantity	Units	Value		
		PF-1A	PF-1B	PF-1C
Young's modulus in radial direction, E_R	GPa	28.0	30.4	37.7
Young's modulus in toroidal direction, E_Θ	GPa	90.2	84.9	90.0
Young's modulus in vertical direction, E_Z	GPa	44.0	28.7	31.6
Shear modulus, G_{RZ}	GPa	8.3	7.0	8.4
Shear modulus, $G_{R\Theta}$	GPa	10.4	10.7	13.3
Shear modulus, $G_{Z\Theta}$	GPa	15.0	10.2	11.5

***Table A1-6**: Coil pack Poisson ratio values*

Quantity	Value		
	PF-1A	PF-1B	PF-1C
Poisson's ratio, ν_{RZ}	.384	.427	.415
Poisson's ratio, $\nu_{R\Theta}$	.341	.345	.345
Poisson's ratio, $\nu_{Z\Theta}$	.347	.344	.342

PF1A Composite Conductor Smeared Mechanical Properties
(Without Cooling Hole)

1.0 Problem Description

To determine the PF1A composite conductor effective monolith/ homogenized/ smeared orthotropic mechanical properties: Elastic Modulus; Shear Modulus; Poisson's Ratio; and Coefficient of Thermal Expansion, using the unit cell method.

Reference: "Effective mechanical properties of EM composite conductors: an analytical and finite element modeling approach", W. Sun, Composite Structures, Volume 58, pg 411-421, 2002

2.0 Given

The details of PF1A composite conductor are shown in Figures 1 thru 4 (note: without the cooling hole).

Metallic Conducting Core Width	$L := 13.2 \cdot mm$
Ratio of Core Height-to-Width	$a := 1.8846$
Metallic Core Height	$aL := a \cdot L = 24.877 \cdot mm$
Metallic Core Young's Modulus	$E_M := 110 \cdot GPa$
Metallic Core Poisson's Ratio	$\nu_M := .34$
Metallic Core Shear Modulus	$G_M := \dfrac{E_M}{2 \cdot (1 + \nu_M)} = 41.045 \cdot GPa$
Phase 1 Insulation Conductor Width-to-Thickness Ratio	$b := .013462$
Phase 1 Insulation Thickness	$bL := b \cdot L = 0.178 \cdot mm$

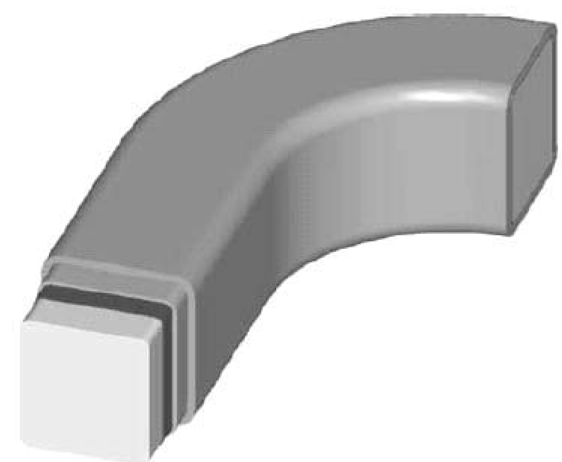


Fig. 1. Schematic view of a conductor structural winding.

Phase 1 Insulation Young's Modulus	$E_{P1} := 2.5 \cdot GPa$
Phase 1 Insulation Poisson's Ratio	$\nu_{P1} := .34$
Phase 1 Insulation Shear Modulus	$G_{P1} := \dfrac{E_{P1}}{2 \cdot (1 + \nu_{P1})} = 0.933 \cdot GPa$
Phase 2 Insulation Conductor Width-to-Thickness Ratio	$c := .013462$
Phase 2 Insulation Thickness	$cL := c \cdot L = 0.178 \cdot mm$
Phase 2 Insulation Young's Modulus	$E_{P2} := 2.5 \cdot GPa$
Phase 2 Insulation Poisson's Ratio	$\nu_{P2} := .34$
Phase 2 InsulationShear Modulus	$G_{P2} := \dfrac{E_{P2}}{2 \cdot (1 + \nu_{P2})} = 0.933 \cdot GPa$
Phase 3 Insulation Conductor Width-to-Thickness Ratio	$d := .051923$
Phase 3 Insulation Thickness	$dL := d \cdot L = 0.685 \cdot mm$
Phase 3 Fabric Weave	Plain
Phase 3 Fiber +/- Angle	$\theta := 22.5 \cdot deg$
Phase 3 Glass Fiber Young's Modulus	$E_f := 87.5 \cdot GPa$
Phase 3 Glass Fiber Poisson's Ratio	$\nu_f := .23$
Phase 3 Glass Fiber Shear Modulus	$G_f := \dfrac{E_f}{2 \cdot (1 + \nu_f)} = 35.569 \cdot GPa$

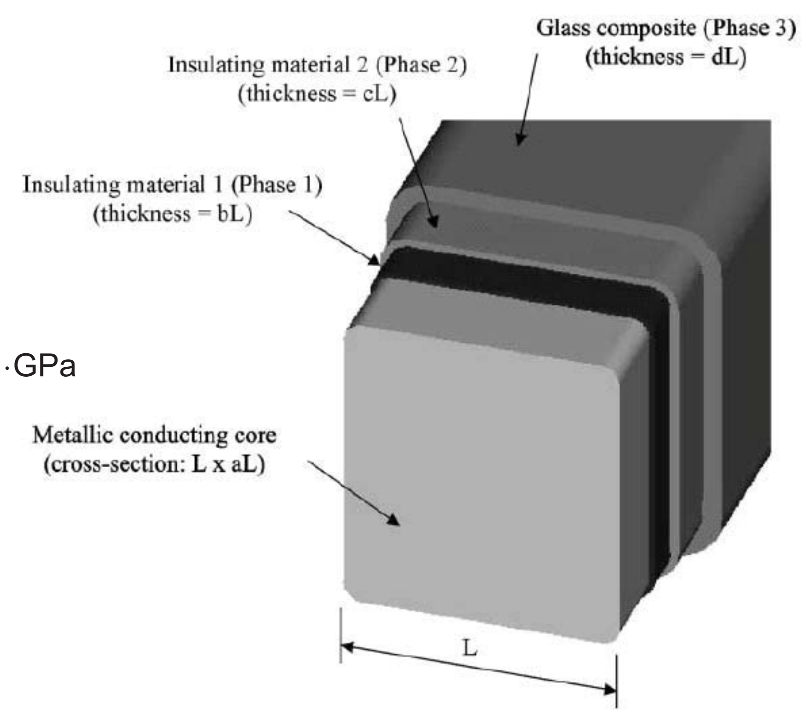


Fig. 2. Schematic representation of a conductor unit cell.

Phase 3 Glass Fiber Volume Fraction $V_f := .5$

Phase 3 Epoxy Young's Modulus $E_m := 4 \cdot GPa$

Phase 3 Epoxy Poisson's Ratio $\nu_m := .36$

Phase 3 Epoxy Shear Modulus $G_m := \frac{E_m}{2 \cdot (1 + \nu_m)} = 1.471 \cdot GPa$

Phase 3 Epoxy Volume Fraction $V_m := .5$

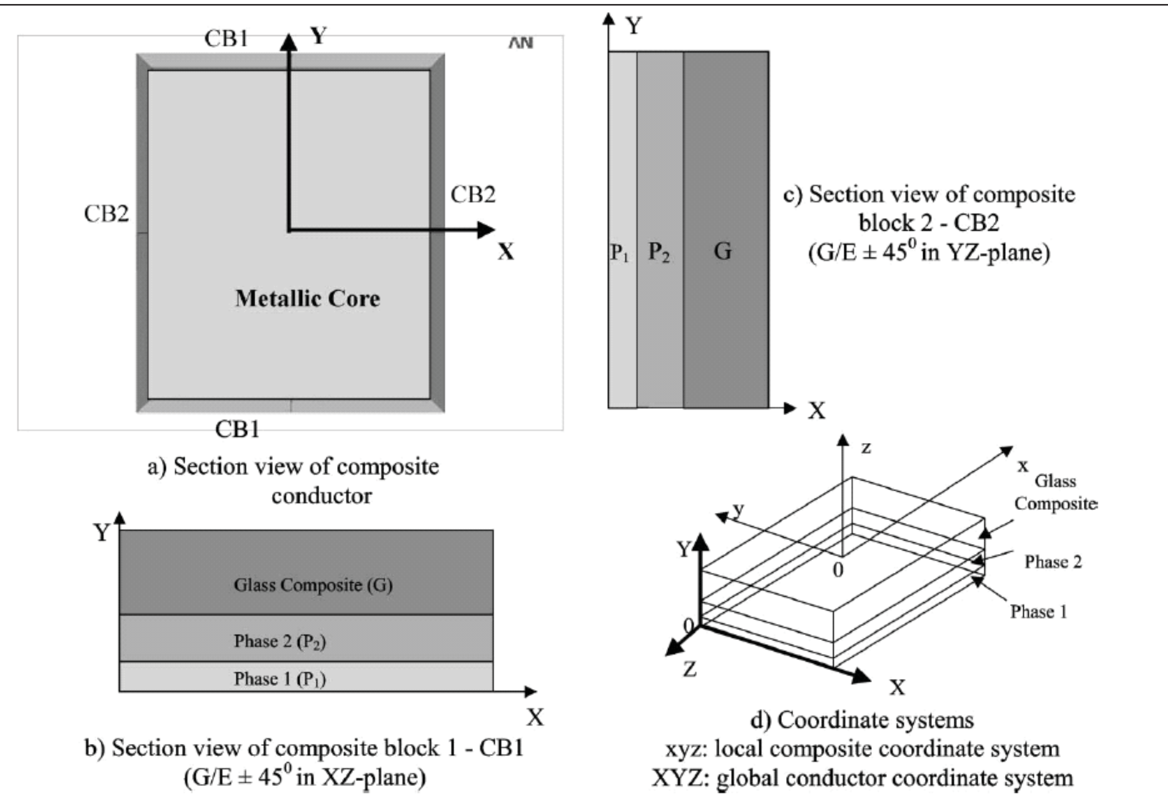
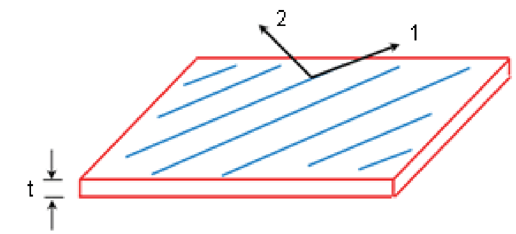


Fig. 4. Section views of conductor and insulation composite blocks. Section view of (a) composite conductor, (b) composite block 1—CB1 ($G/E \pm 45°$ in XZ-plane), (c) composite block 2—CB2 ($G/E \pm 45°$ in YZ-plane), and (d) coordinate systems, xyz: local composite coordinate system; XYZ: global conductor coordinate system.

3.0 Elastic Constants of Woven Glass/ Epoxy Composite Laminae in Local (x,y,z) Coordinate System

3.1 The in-plane elastic constants in the fiber (1, 2, 3) coordinate system



The in-plane Elastic Modulus along the axis (longitudinal direction) of the fiber is given by:

$$E_{11} := V_f \cdot E_f + V_m \cdot E_m = 45.75 \cdot GPa$$

The Poisson's Ratio is given by:

$$\nu_{12} := V_f \cdot \nu_f + V_m \cdot \nu_m = 0.295$$

The Shear Modulus is given by:

$$G_{12} := G_m \cdot \frac{(G_f + G_m) + V_f \cdot (G_f - G_m)}{(G_f + G_m) - V_f \cdot (G_f - G_m)} = 3.979 \cdot GPa$$

The in-plane Elastic Modulus normal to the axis (lateral direction) to the fiber is given by:

$$E_{22} := \frac{\left(V_f + V_m \cdot \frac{E_m}{E_f}\right) \cdot \left[V_f + V_m \cdot .5 \cdot \left(1 + \frac{E_m}{E_f}\right)\right]}{\left(V_f + V_m \cdot \frac{E_m}{E_f}\right) \cdot \left[\frac{V_f}{E_f} + .5 \cdot \left(1 + \frac{E_m}{E_f}\right) \cdot \frac{V_m}{E_m}\right] + V_f \cdot V_m \cdot \left(\frac{-\nu_m}{E_m} + \frac{\nu_f}{E_f}\right) \cdot \frac{\left(\frac{-\nu_f}{E_f} + \frac{\nu_m}{E_m}\right) \cdot \left[\frac{E_m}{E_f} - .5 \cdot \left(1 + \frac{E_m}{E_f}\right)\right]}{\left(\frac{1}{E_f} - \frac{1}{E_m}\right)}} = 11.94 \cdot GPa$$

The Shear Modulus is given by:

$$G_{23} := \frac{.5\cdot\left[V_f + V_m\cdot.5\cdot\left(1+\frac{E_m}{E_f}\right)\right]}{\left[V_f\cdot\left(\frac{1}{E_f}+\frac{\nu_f}{E_f}\right)+V_m\cdot.5\cdot\left(1+\frac{E_m}{E_f}\right)\cdot\left(\frac{1}{E_m}+\frac{\nu_m}{E_m}\right)\right]} = 3.969\cdot GPa$$

The Poisson's Ratio is given by:

$$\nu_{23} := \nu_{12} = 0.295$$

The out-of-plane Elastic Modulus (transverse direction) is given by:

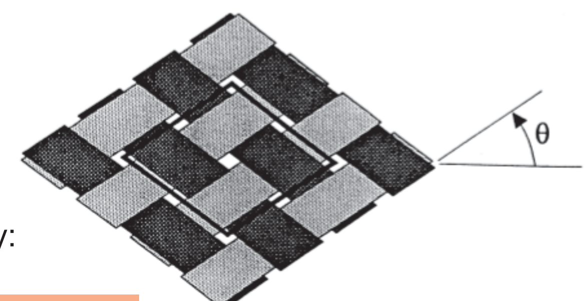
$$E_{33} := \frac{E_f\cdot E_m}{V_f\cdot E_m + V_m\cdot E_f} = 7.65\cdot GPa$$

The Shear Modulus is given by:

$$G_{13} := G_{23} = 3.969\cdot GPa$$

The Poisson's Ratio is given by:

$$\nu_{13} := \nu_{12} = 0.295$$



3.2 The in-plane elastic constants in the local (x, y, z) coordinate system

The in-plane Elastic Modulus of the glass-epoxy laminae in the local x direction is given by:

$$E_{G\_xx} := \frac{E_{11}\cdot E_{22}}{E_{22}\cdot\cos(\theta)^4 + E_{11}\cdot E_{22}\cdot\left(\frac{1}{G_{12}}-\frac{2\cdot\nu_{12}}{E_{11}}\right)\cdot\sin(\theta)^2\cdot\cos(\theta)^2 + E_{11}\cdot\sin(\theta)^4} = 21.042\cdot GPa$$

The in-plane Elastic Modulus of the glass-epoxy laminae in the local y direction is given by:

$$E_{G\_yy} := \frac{E_{11} \cdot E_{22}}{E_{11} \cdot \cos(\theta)^4 + E_{11} \cdot E_{22} \cdot \left(\frac{1}{G_{12}} - \frac{2 \cdot \nu_{12}}{E_{11}}\right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2 + E_{22} \cdot \sin(\theta)^4} = 10.954 \cdot \mathrm{GPa}$$

The Poisson's Ratio is given by:

$$\nu_{G\_xy} := E_{G\_xx} \cdot \left[\frac{\nu_{12}}{E_{11}} \cdot \left(\sin(\theta)^4 + \cos(\theta)^4\right) - \left(\frac{1}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}}\right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2\right] = 0.485$$

The Shear Modulus is given by:

$$G_{G\_xy} := \frac{G_{12}}{2 \cdot G_{12} \cdot \left(\frac{2}{E_{11}} + \frac{2}{E_{22}} + \frac{4 \cdot \nu_{12}}{E_{11}} - \frac{1}{G_{12}}\right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2 + \left(\sin(\theta)^4 + \cos(\theta)^4\right)} = 5.408 \cdot \mathrm{GPa}$$

The Elastic Modulus in the local z direction is given by:

$$E_{G\_zz} := E_{33} = 7.65 \cdot \mathrm{GPa}$$

And the remaining constants are given by:

$$G_{G\_yz} := G_{23} = 3.969 \cdot \mathrm{GPa}$$

$$G_{G\_xz} := G_{13} = 3.969 \cdot \mathrm{GPa}$$

$$\nu_{G\_yz} := \nu_{23} = 0.295$$

$$\nu_{G\_xz} := \nu_{13} = 0.295$$

4.0 Elastic Constants of Insulation Composite Laminae in the Global (X,Y,Z) Coordinate System

4.1 Elastic constants of insulation composite CB1 laminae in the global (X,Y,Z) coordinate system

The volume fraction each of the 3 phases in the CB1 laminae are given by:

Volume Fraction of Phase 1 $$V_{1\_P1} := \frac{b}{(b+c+d)} = 0.171$$

Volume Fraction of Phase 2 $$V_{1\_P2} := \frac{c}{(b+c+d)} = 0.171$$

Volume Fraction of Phase 3 $$V_{1\_G} := \frac{d}{(b+c+d)} = 0.659$$

The elastic constants for composite block CB1 are given by:

$$E_{CB1\_XX} := V_{1\_P1} \cdot E_{P1} + V_{1\_P2} \cdot E_{P2} + V_{1\_G} \cdot E_{G\_yy} = 8.067 \cdot GPa$$

$$E_{CB1\_YY} := \frac{E_{P1} \cdot E_{P2} \cdot E_{G\_zz}}{V_{1\_P1} \cdot E_{P2} \cdot E_{G\_zz} + V_{1\_P2} \cdot E_{P1} \cdot E_{G\_zz} + V_{1\_G} \cdot E_{P1} \cdot E_{P2}} = 4.491 \cdot GPa$$

$$E_{CB1\_ZZ} := V_{1\_P1} \cdot E_{P1} + V_{1\_P2} \cdot E_{P2} + V_{1\_G} \cdot E_{G\_xx} = 14.711 \cdot GPa$$

$$\nu_{CB1\_XY} := V_{1\_P1} \cdot \nu_{P1} + V_{1\_P2} \cdot \nu_{P2} + V_{1\_G} \cdot \nu_{G\_yz} = 0.31$$

$$G_{CB1\_XY} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_yz}}{V_{1\_P1} \cdot G_{P2} \cdot G_{G\_yz} + V_{1\_P2} \cdot G_{P1} \cdot G_{G\_yz} + V_{1\_G} \cdot G_{P2} \cdot G_{P1}} = 1.88 \cdot GPa$$

$$G_{CB1\_YZ} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_xz}}{V_{1\_P1} \cdot G_{P2} \cdot G_{G\_xz} + V_{1\_P2} \cdot G_{P1} \cdot G_{G\_xz} + V_{1\_G} \cdot G_{P2} \cdot G_{P1}} = 1.88 \cdot GPa$$

$$G_{CB1\_XZ} := V_{1\_P1} \cdot G_{P1} + V_{1\_P2} \cdot G_{P2} + V_{1\_G} \cdot G_{G\_xy} = 3.88 \cdot GPa$$

$$\nu_{CB1\_YZ} := V_{1\_P1} \cdot \nu_{P1} + V_{1\_P2} \cdot \nu_{P2} + V_{1\_G} \cdot \nu_{G\_xz} = 0.31$$

$$\nu_{CB1\_XZ} := \frac{\nu_{P1} \cdot \nu_{P2} \cdot \nu_{G\_xy}}{V_{1\_P1} \cdot \nu_{P2} \cdot \nu_{G\_xy} + V_{1\_P2} \cdot \nu_{P1} \cdot \nu_{G\_xy} + V_{1\_G} \cdot \nu_{P1} \cdot \nu_{P2}} = 0.423$$

4.2 Elastic constants of insulation composite CB2 laminae in the global (X,Y,Z) coordinate system

The volume fraction each of the 3 phases in the CB1 laminae are given by:

Volume Fraction of Phase 1 $$V_{2\_P1} := \frac{b}{(b + c + d)} = 0.171$$

Volume Fraction of Phase 2 $$V_{2\_P2} := \frac{c}{(b + c + d)} = 0.171$$

Volume Fraction of Phase 3 $$V_{2\_G} := \frac{d}{(b + c + d)} = 0.659$$

The elastic constants for composite block CB2 are given by:

$$E_{CB2\_XX} := \frac{E_{P1} \cdot E_{P2} \cdot E_{G\_zz}}{V_{2\_P1} \cdot E_{P2} \cdot E_{G\_zz} + V_{2\_P2} \cdot E_{P1} \cdot E_{G\_zz} + V_{2\_G} \cdot E_{P1} \cdot E_{P2}} = 4.491 \cdot GPa$$

$$E_{CB2\_YY} := V_{2\_P1} \cdot E_{P1} + V_{2\_P2} \cdot E_{P2} + V_{2\_G} \cdot E_{G\_yy} = 8.067 \cdot GPa$$

$$E_{CB2\_ZZ} := V_{2\_P1} \cdot E_{P1} + V_{2\_P2} \cdot E_{P2} + V_{2\_G} \cdot E_{G\_xx} = 14.711 \cdot GPa$$

$$\nu_{CB2\_XY} := V_{2\_P1} \cdot \nu_{P1} + V_{2\_P2} \cdot \nu_{P2} + V_{2\_G} \cdot \nu_{G\_yz} = 0.31$$

$$G_{CB2\_XY} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_yz}}{V_{2\_P1} \cdot G_{P2} \cdot G_{G\_yz} + V_{2\_P2} \cdot G_{P1} \cdot G_{G\_yz} + V_{2\_G} \cdot G_{P1} \cdot G_{P2}} = 1.88 \cdot GPa$$

$$G_{CB2\_YZ} := V_{2\_P1} \cdot G_{P1} + V_{2\_P2} \cdot G_{P2} + V_{2\_G} \cdot G_{G\_xy} = 3.88 \cdot GPa$$

$$G_{CB2\_XZ} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_xz}}{V_{2\_P1} \cdot G_{P2} \cdot G_{G\_xz} + V_{2\_P2} \cdot G_{P1} \cdot G_{G\_xz} + V_{2\_G} \cdot G_{P1} \cdot G_{P2}} = 1.88 \cdot GPa$$

$$\nu_{CB2\_YZ} := \frac{\nu_{P1} \cdot \nu_{P2} \cdot \nu_{G\_xy}}{V_{2\_P1} \cdot \nu_{P2} \cdot \nu_{G\_xy} + V_{2\_P2} \cdot \nu_{P1} \cdot \nu_{G\_xy} + V_{2\_G} \cdot \nu_{P1} \cdot \nu_{P2}} = 0.423$$

$$\nu_{CB2\_XZ} := V_{2\_P1} \cdot \nu_{P1} + V_{2\_P2} \cdot \nu_{P2} + V_{2\_G} \cdot \nu_{G\_xz} = 0.31$$

5.0 Elastic Constants of Insulation Composite Conductor in the Global (X,Y,Z) Coordinate System

5.1 Volume fraction of the composite conductor components

The overall width of the composite conductor is given by:

$$L_X := L + 2 \cdot L \cdot (b + c + d) = 15.282 \cdot mm$$

The average width for CB1 is

$$L_{X\_CB1} := L + L \cdot (b + c + d) = 14.241 \cdot mm$$

The overall height of the composite conductor is given by:

$$L_Y := aL + 2 \cdot L \cdot (b + c + d) = 26.958 \cdot mm$$

The average height for CB2 is

$$L_{Y\_CB2} := aL + L \cdot (b + c + d) = 25.918 \cdot mm$$

The total cross sectional area of the composite conductor is given by:

$$A_{ZZ} := L_X \cdot L_Y = 411.965 \cdot mm^2$$

The cross sectional area of the metal core is given by:

$$A_M := L \cdot aL = 328.373 \cdot mm^2$$

The cross sectional area of CB1 is given by:

$$A_{CB1} := 2 \cdot L_{X\_CB1} \cdot L \cdot (b + c + d) = 29.643 \cdot mm^2$$

The cross sectional area of CB2 is given by:

$$A_{CB2} := 2 \cdot L_{Y\_CB2} \cdot L \cdot (b + c + d) = 53.949 \cdot mm^2$$

The volume fraction each of the 3 components of in the composite conductor are given by:

Volume Fraction of Metal Core $$V_M := \frac{A_M}{A_{ZZ}} = 0.797$$

Volume Fraction of CB1 $$V_{CB1} := \frac{A_{CB1}}{A_{ZZ}} = 0.072$$

Volume Fraction of CB2 $$V_{CB2} := \frac{A_{CB2}}{A_{ZZ}} = 0.131$$

5.2 Elastic constants for the composite conductor

The elastic constants for the composite conductor are given by:

$$E_{CC\_XX} := V_{CB1} \cdot E_{CB1\_XX} + \frac{E_M \cdot E_{CB2\_XX}}{V_{CB2} \cdot E_M + V_M \cdot E_{CB2\_XX}} = 28.049 \cdot GPa$$

$$E_{CC\_YY} := V_{CB2} \cdot E_{CB2\_YY} + \frac{E_M \cdot E_{CB1\_YY}}{V_{CB1} \cdot E_M + V_M \cdot E_{CB1\_YY}} = 44.033 \cdot GPa$$

$$E_{CC\_ZZ} := V_M \cdot E_M + V_{CB1} \cdot E_{CB1\_XX} + V_{CB2} \cdot E_{CB2\_ZZ} = 90.187 \cdot GPa$$

$$G_{CC\_XY} := \frac{G_{CB1\_XY} \cdot G_{CB2\_XY} \cdot G_M}{G_{CB2\_XY} \cdot G_M \cdot V_{CB1} + \left(V_{CB2} + V_M\right) \cdot G_{CB1\_XY} \cdot \left(V_{CB2} \cdot G_M + V_M \cdot G_{CB2\_XY}\right)} = 8.268 \cdot GPa$$

$$G_{CC\_YZ} := \frac{G_{CB1\_YZ} \cdot \left(V_{CB1} \cdot G_{CB2\_YZ} + V_M \cdot G_M\right)}{\left(V_{CB2} + V_M\right) \cdot G_{CB1\_YZ} + V_{CB1} \cdot \left(V_{CB2} \cdot G_{CB2\_YZ} + V_M \cdot G_M\right)} = 14.999 \cdot GPa$$

$$G_{CC\_XZ} := \frac{G_{CB2\_XZ} \cdot \left(V_{CB1} \cdot G_{CB1\_XZ} + V_M \cdot G_M\right)}{\left(V_{CB1} + V_M\right) \cdot G_{CB2\_XZ} + V_{CB2} \cdot \left(V_{CB2} \cdot G_{CB1\_XZ} + V_M \cdot G_M\right)} = 10.364 \cdot GPa$$

$$\nu_{CC\_XY} := V_{CB1} \cdot \nu_{CB1\_XY} + \frac{\nu_M \cdot \nu_{CB2\_XY}}{V_{CB2} \cdot \nu_M + V_M \cdot \nu_{CB2\_XY}} = 0.384$$

$$\nu_{CC\_YZ} := \frac{\nu_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}}{V_{CB1} \cdot \nu_{CB2\_YZ} \cdot \nu_M + V_{CB2} \cdot \nu_{CB1\_YZ} \cdot \nu_M + V_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}} = 0.347$$

$$\nu_{CC\_XZ} := \frac{\nu_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}}{V_{CB1} \cdot \nu_{CB2\_XZ} \cdot \nu_M + V_{CB2} \cdot \nu_{CB1\_XZ} \cdot \nu_M + V_M \cdot \nu_{CB1\_XZ} \cdot \nu_{CB2\_XZ}} = 0.341$$

PF1B Composite Conductor Smeared Mechanical Properties
(Without Cooling Hole)

1.0 Problem Description

To determine the PF1B composite conductor effective monolith/ homogenized/ smeared orthotropic mechanical properties: Elastic Modulus; Shear Modulus; Poisson's Ratio; and Coefficient of Thermal Expansion, using the unit cell method.

Reference: "Effective mechanical properties of EM composite conductors: an analytical and finite element modeling approach", W. Sun, Composite Structures, Volume 58, pg 411-421, 2002

2.0 Given

The details of PF1B composite conductor are shown in Figures 1 thru 4 (note: without the cooling hole).

Metallic Conducting Core Width $L := 13.716 \cdot mm$

Ratio of Core Height-to-Width $a := .9259$

Metallic Core Height $aL := a \cdot L = 12.7 \cdot mm$

Metallic Core Young's Modulus $E_M := 110 \cdot GPa$

Metallic Core Poisson's Ratio $\nu_M := .34$

Metallic Core Shear Modulus $G_M := \dfrac{E_M}{2 \cdot (1 + \nu_M)} = 41.045 \cdot GPa$

Phase 1 Insulation Conductor Width-to-Thickness Ratio $b := .012963$

Phase 1 Insulation Thickness $bL := b \cdot L = 0.178 \cdot mm$

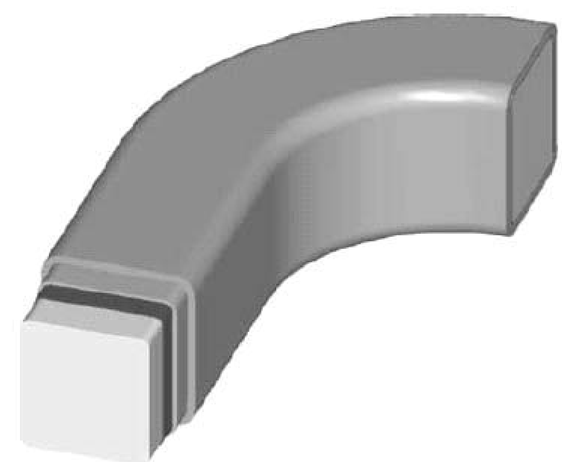


Fig. 1. Schematic view of a conductor structural winding.

Phase 1 Insulation Young's Modulus	$E_{P1} := 2.5 \cdot GPa$	
Phase 1 Insulation Poisson's Ratio	$\nu_{P1} := .34$	
Phase 1 Insulation Shear Modulus	$G_{P1} := \dfrac{E_{P1}}{2 \cdot (1 + \nu_{P1})} = 0.933 \cdot GPa$	
Phase 2 Insulation Conductor Width-to-Thickness Ratio	$c := .012963$	
Phase 2 Insulation Thickness	$cL := c \cdot L = 0.178 \cdot mm$	
Phase 2 Insulation Young's Modulus	$E_{P2} := 2.5 \cdot GPa$	
Phase 2 Insulation Poisson's Ratio	$\nu_{P2} := .34$	
Phase 2 InsulationShear Modulus	$G_{P2} := \dfrac{E_{P2}}{2 \cdot (1 + \nu_{P2})} = 0.933 \cdot GPa$	
Phase 3 Insulation Conductor Width-to-Thickness Ratio	$d := .05$	
Phase 3 Insulation Thickness	$dL := d \cdot L = 0.686 \cdot mm$	
Phase 3 Fabric Weave	Plain	
Phase 3 Fiber +/- Angle	$\theta := 22.5 \cdot deg$	
Phase 3 Glass Fiber Young's Modulus	$E_f := 87.5 \cdot GPa$	
Phase 3 Glass Fiber Poisson's Ratio	$\nu_f := .23$	
Phase 3 Glass Fiber Shear Modulus	$G_f := \dfrac{E_f}{2 \cdot (1 + \nu_f)} = 35.569 \cdot GPa$	

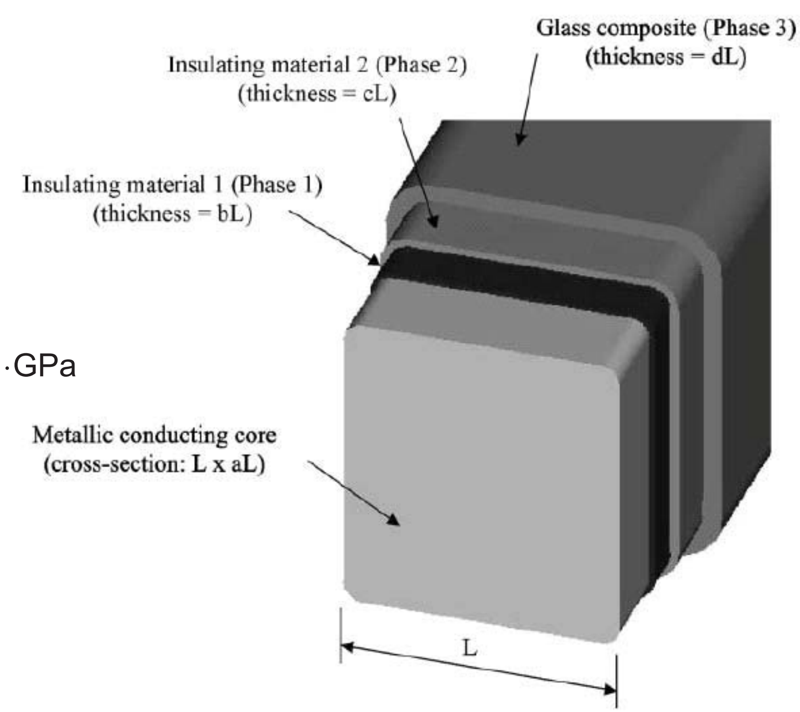


Fig. 2. Schematic representation of a conductor unit cell.

Phase 3 Glass Fiber Volume Fraction $V_f := .5$

Phase 3 Epoxy Young's Modulus $E_m := 4 \cdot GPa$

Phase 3 Epoxy Poisson's Ratio $\nu_m := .36$

Phase 3 Epoxy Shear Modulus $G_m := \dfrac{E_m}{2 \cdot (1 + \nu_m)} = 1.471 \cdot GPa$

Phase 3 Epoxy Volume Fraction $V_m := .5$

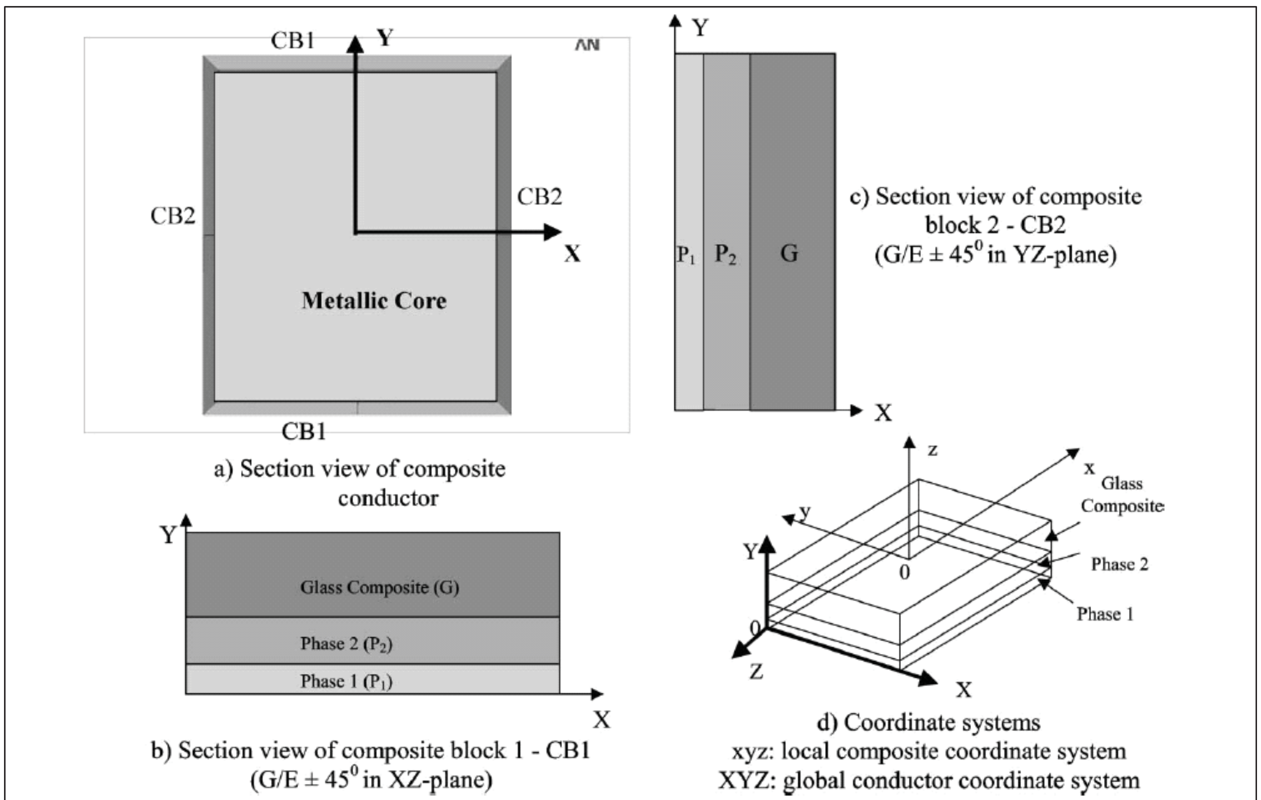
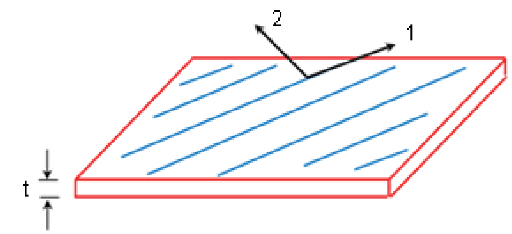


Fig. 4. Section views of conductor and insulation composite blocks. Section view of (a) composite conductor, (b) composite block 1—CB1 ($G/E \pm 45°$ in XZ-plane), (c) composite block 2—CB2 ($G/E \pm 45°$ in YZ-plane), and (d) coordinate systems, xyz: local composite coordinate system; XYZ: global conductor coordinate system.

3.0 Elastic Constants of Woven Glass/ Epoxy Composite Laminae in Local (x,y,z) Coordinate System

3.1 The in-plane elastic constants in the fiber (1, 2, 3) coordinate system

The in-plane Elastic Modulus along the axis (longitudinal direction) of the fiber is given by:



$$E_{11} := V_f \cdot E_f + V_m \cdot E_m = 45.75 \cdot \text{GPa}$$

The Poisson's Ratio is given by:

$$\nu_{12} := V_f \cdot \nu_f + V_m \cdot \nu_m = 0.295$$

The Shear Modulus is given by:

$$G_{12} := G_m \cdot \frac{(G_f + G_m) + V_f \cdot (G_f - G_m)}{(G_f + G_m) - V_f \cdot (G_f - G_m)} = 3.979 \cdot \text{GPa}$$

The in-plane Elastic Modulus normal to the axis (lateral direction) to the fiber is given by:

$$E_{22} := \frac{\left(V_f + V_m \cdot \frac{E_m}{E_f}\right) \cdot \left[V_f + V_m \cdot .5 \cdot \left(1 + \frac{E_m}{E_f}\right)\right]}{\left(V_f + V_m \cdot \frac{E_m}{E_f}\right) \cdot \left[\frac{V_f}{E_f} + .5 \cdot \left(1 + \frac{E_m}{E_f}\right) \cdot \frac{V_m}{E_m}\right] + V_f \cdot V_m \cdot \left(\frac{-\nu_m}{E_m} + \frac{\nu_f}{E_f}\right) \cdot \frac{\left(\frac{-\nu_f}{E_f} + \frac{\nu_m}{E_m}\right) \cdot \left[\frac{E_m}{E_f} - .5 \cdot \left(1 + \frac{E_m}{E_f}\right)\right]}{\left(\frac{1}{E_f} - \frac{1}{E_m}\right)}} = 11.94 \cdot \text{GPa}$$

The Shear Modulus is given by:

$$G_{23} := \frac{.5 \cdot \left[V_f + V_m \cdot .5 \cdot \left(1 + \frac{E_m}{E_f} \right) \right]}{\left[V_f \cdot \left(\frac{1}{E_f} + \frac{\nu_f}{E_f} \right) + V_m \cdot .5 \cdot \left(1 + \frac{E_m}{E_f} \right) \cdot \left(\frac{1}{E_m} + \frac{\nu_m}{E_m} \right) \right]} = 3.969 \cdot GPa$$

The Poisson's Ratio is given by:

$$\nu_{23} := \nu_{12} = 0.295$$

The out-of-plane Elastic Modulus (transverse direction) is given by:

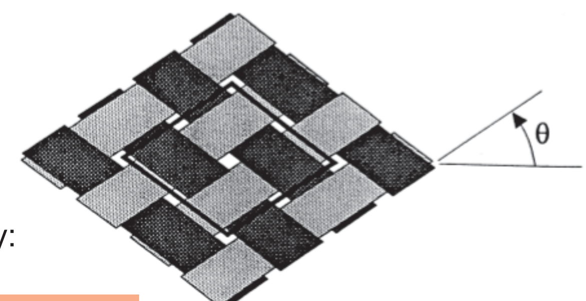
$$E_{33} := \frac{E_f \cdot E_m}{V_f \cdot E_m + V_m \cdot E_f} = 7.65 \cdot GPa$$

The Shear Modulus is given by:

$$G_{13} := G_{23} = 3.969 \cdot GPa$$

The Poisson's Ratio is given by:

$$\nu_{13} := \nu_{12} = 0.295$$



3.2 The in-plane elastic constants in the local (x, y, z) coordinate system

The in-plane Elastic Modulus of the glass-epoxy laminae in the local x direction is given by:

$$E_{G\_xx} := \frac{E_{11} \cdot E_{22}}{E_{22} \cdot \cos(\theta)^4 + E_{11} \cdot E_{22} \cdot \left(\frac{1}{G_{12}} - \frac{2 \cdot \nu_{12}}{E_{11}} \right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2 + E_{11} \cdot \sin(\theta)^4} = 21.042 \cdot GPa$$

The in-plane Elastic Modulus of the glass-epoxy laminae in the local y direction is given by:

$$E_{G\_yy} := \frac{E_{11} \cdot E_{22}}{E_{11} \cdot \cos(\theta)^4 + E_{11} \cdot E_{22} \cdot \left(\frac{1}{G_{12}} - \frac{2 \cdot \nu_{12}}{E_{11}}\right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2 + E_{22} \cdot \sin(\theta)^4} = 10.954 \cdot \mathrm{GPa}$$

The Poisson's Ratio is given by:

$$\nu_{G\_xy} := E_{G\_xx} \cdot \left[\frac{\nu_{12}}{E_{11}} \cdot \left(\sin(\theta)^4 + \cos(\theta)^4\right) - \left(\frac{1}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}}\right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2\right] = 0.485$$

The Shear Modulus is given by:

$$G_{G\_xy} := \frac{G_{12}}{2 \cdot G_{12} \cdot \left(\frac{2}{E_{11}} + \frac{2}{E_{22}} + \frac{4 \cdot \nu_{12}}{E_{11}} - \frac{1}{G_{12}}\right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2 + \left(\sin(\theta)^4 + \cos(\theta)^4\right)} = 5.408 \cdot \mathrm{GPa}$$

The Elastic Modulus in the local z direction is given by:

$$E_{G\_zz} := E_{33} = 7.65 \cdot \mathrm{GPa}$$

And the remaining constants are given by:

$$G_{G\_yz} := G_{23} = 3.969 \cdot \mathrm{GPa}$$

$$G_{G\_xz} := G_{13} = 3.969 \cdot \mathrm{GPa}$$

$$\nu_{G\_yz} := \nu_{23} = 0.295$$

$$\nu_{G\_xz} := \nu_{13} = 0.295$$

4.0 Elastic Constants of Insulation Composite Laminae in the Global (X,Y,Z) Coordinate System

4.1 Elastic constants of insulation composite CB1 laminae in the global (X,Y,Z) coordinate system

The volume fraction each of the 3 phases in the CB1 laminae are given by:

Volume Fraction of Phase 1 $$V_{1\_P1} := \frac{b}{(b+c+d)} = 0.171$$

Volume Fraction of Phase 2 $$V_{1\_P2} := \frac{c}{(b+c+d)} = 0.171$$

Volume Fraction of Phase 3 $$V_{1\_G} := \frac{d}{(b+c+d)} = 0.659$$

The elastic constants for composite block CB1 are given by:

$$E_{CB1\_XX} := V_{1\_P1} \cdot E_{P1} + V_{1\_P2} \cdot E_{P2} + V_{1\_G} \cdot E_{G\_yy} = 8.067 \cdot GPa$$

$$E_{CB1\_YY} := \frac{E_{P1} \cdot E_{P2} \cdot E_{G\_zz}}{V_{1\_P1} \cdot E_{P2} \cdot E_{G\_zz} + V_{1\_P2} \cdot E_{P1} \cdot E_{G\_zz} + V_{1\_G} \cdot E_{P1} \cdot E_{P2}} = 4.491 \cdot GPa$$

$$E_{CB1\_ZZ} := V_{1\_P1} \cdot E_{P1} + V_{1\_P2} \cdot E_{P2} + V_{1\_G} \cdot E_{G\_xx} = 14.711 \cdot GPa$$

$$\nu_{CB1\_XY} := V_{1\_P1} \cdot \nu_{P1} + V_{1\_P2} \cdot \nu_{P2} + V_{1\_G} \cdot \nu_{G\_yz} = 0.31$$

$$G_{CB1\_XY} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_yz}}{V_{1\_P1} \cdot G_{P2} \cdot G_{G\_yz} + V_{1\_P2} \cdot G_{P1} \cdot G_{G\_yz} + V_{1\_G} \cdot G_{P2} \cdot G_{P1}} = 1.88 \cdot GPa$$

$$G_{CB1\_YZ} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_xz}}{V_{1\_P1} \cdot G_{P2} \cdot G_{G\_xz} + V_{1\_P2} \cdot G_{P1} \cdot G_{G\_xz} + V_{1\_G} \cdot G_{P2} \cdot G_{P1}} = 1.88 \cdot GPa$$

$$G_{CB1\_XZ} := V_{1\_P1} \cdot G_{P1} + V_{1\_P2} \cdot G_{P2} + V_{1\_G} \cdot G_{G\_xy} = 3.88 \cdot GPa$$

$$\nu_{CB1\_YZ} := V_{1\_P1} \cdot \nu_{P1} + V_{1\_P2} \cdot \nu_{P2} + V_{1\_G} \cdot \nu_{G\_xz} = 0.31$$

$$\nu_{CB1\_XZ} := \frac{\nu_{P1} \cdot \nu_{P2} \cdot \nu_{G\_xy}}{V_{1\_P1} \cdot \nu_{P2} \cdot \nu_{G\_xy} + V_{1\_P2} \cdot \nu_{P1} \cdot \nu_{G\_xy} + V_{1\_G} \cdot \nu_{P1} \cdot \nu_{P2}} = 0.423$$

4.2 Elastic constants of insulation composite CB2 laminae in the global (X,Y,Z) coordinate system

The volume fraction each of the 3 phases in the CB1 laminae are given by:

Volume Fraction of Phase 1 $$V_{2\_P1} := \frac{b}{(b + c + d)} = 0.171$$

Volume Fraction of Phase 2 $$V_{2\_P2} := \frac{c}{(b + c + d)} = 0.171$$

Volume Fraction of Phase 3 $$V_{2\_G} := \frac{d}{(b + c + d)} = 0.659$$

The elastic constants for composite block CB2 are given by:

$$E_{CB2\_XX} := \frac{E_{P1} \cdot E_{P2} \cdot E_{G\_zz}}{V_{2\_P1} \cdot E_{P2} \cdot E_{G\_zz} + V_{2\_P2} \cdot E_{P1} \cdot E_{G\_zz} + V_{2\_G} \cdot E_{P1} \cdot E_{P2}} = 4.491 \cdot GPa$$

$$E_{CB2\_YY} := V_{2\_P1} \cdot E_{P1} + V_{2\_P2} \cdot E_{P2} + V_{2\_G} \cdot E_{G\_yy} = 8.067 \cdot GPa$$

$$E_{CB2\_ZZ} := V_{2\_P1} \cdot E_{P1} + V_{2\_P2} \cdot E_{P2} + V_{2\_G} \cdot E_{G\_xx} = 14.711 \cdot GPa$$

$$\nu_{CB2\_XY} := V_{2\_P1} \cdot \nu_{P1} + V_{2\_P2} \cdot \nu_{P2} + V_{2\_G} \cdot \nu_{G\_yz} = 0.31$$

$$G_{CB2\_XY} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_yz}}{V_{2\_P1} \cdot G_{P2} \cdot G_{G\_yz} + V_{2\_P2} \cdot G_{P1} \cdot G_{G\_yz} + V_{2\_G} \cdot G_{P1} \cdot G_{P2}} = 1.88 \cdot GPa$$

$$G_{CB2\_YZ} := V_{2\_P1} \cdot G_{P1} + V_{2\_P2} \cdot G_{P2} + V_{2\_G} \cdot G_{G\_xy} = 3.88 \cdot GPa$$

$$G_{CB2\_XZ} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_xz}}{V_{2\_P1} \cdot G_{P2} \cdot G_{G\_xz} + V_{2\_P2} \cdot G_{P1} \cdot G_{G\_xz} + V_{2\_G} \cdot G_{P1} \cdot G_{P2}} = 1.88 \cdot GPa$$

$$\nu_{CB2\_YZ} := \frac{\nu_{P1} \cdot \nu_{P2} \cdot \nu_{G\_xy}}{V_{2\_P1} \cdot \nu_{P2} \cdot \nu_{G\_xy} + V_{2\_P2} \cdot \nu_{P1} \cdot \nu_{G\_xy} + V_{2\_G} \cdot \nu_{P1} \cdot \nu_{P2}} = 0.423$$

$$\nu_{CB2\_XZ} := V_{2\_P1} \cdot \nu_{P1} + V_{2\_P2} \cdot \nu_{P2} + V_{2\_G} \cdot \nu_{G\_xz} = 0.31$$

5.0 Elastic Constants of Insulation Composite Conductor in the Global (X,Y,Z) Coordinate System

5.1 Volume fraction of the composite conductor components

The overall width of the composite conductor is given by:

$$L_X := L + 2 \cdot L \cdot (b + c + d) = 15.799 \cdot mm$$

The average width for CB1 is

$$L_{X\_CB1} := L + L \cdot (b + c + d) = 14.757 \cdot mm$$

The overall height of the composite conductor is given by:

$$L_Y := aL + 2 \cdot L \cdot (b + c + d) = 14.782 \cdot mm$$

The average height for CB2 is

$$L_{Y\_CB2} := aL + L \cdot (b + c + d) = 13.741 \cdot mm$$

The total cross sectional area of the composite conductor is given by:

$$A_{ZZ} := L_X \cdot L_Y = 233.545 \cdot mm^2$$

The cross sectional area of the metal core is given by:

$$A_M := L \cdot aL = 174.188 \cdot mm^2$$

The cross sectional area of CB1 is given by:

$$A_{CB1} := 2 \cdot L_{X\_CB1} \cdot L \cdot (b + c + d) = 30.737 \cdot mm^2$$

The cross sectional area of CB2 is given by:

$$A_{CB2} := 2 \cdot L_{Y\_CB2} \cdot L \cdot (b + c + d) = 28.62 \cdot mm^2$$

The volume fraction each of the 3 components of in the composite conductor are given by:

Volume Fraction of Metal Core $$V_M := \frac{A_M}{A_{ZZ}} = 0.746$$

Volume Fraction of CB1 $$V_{CB1} := \frac{A_{CB1}}{A_{ZZ}} = 0.132$$

Volume Fraction of CB2 $$V_{CB2} := \frac{A_{CB2}}{A_{ZZ}} = 0.123$$

5.2 Elastic constants for the composite conductor

The elastic constants for the composite conductor are given by:

$$E_{CC\_XX} := V_{CB1} \cdot E_{CB1\_XX} + \frac{E_M \cdot E_{CB2\_XX}}{V_{CB2} \cdot E_M + V_M \cdot E_{CB2\_XX}} = 30.416 \cdot GPa$$

$$E_{CC\_YY} := V_{CB2} \cdot E_{CB2\_YY} + \frac{E_M \cdot E_{CB1\_YY}}{V_{CB1} \cdot E_M + V_M \cdot E_{CB1\_YY}} = 28.701 \cdot GPa$$

$$E_{CC\_ZZ} := V_M \cdot E_M + V_{CB1} \cdot E_{CB1\_XX} + V_{CB2} \cdot E_{CB2\_ZZ} = 84.907 \cdot GPa$$

$$G_{CC\_XY} := \frac{G_{CB1\_XY} \cdot G_{CB2\_XY} \cdot G_M}{G_{CB2\_XY} \cdot G_M \cdot V_{CB1} + \left(V_{CB2} + V_M\right) \cdot G_{CB1\_XY} \cdot \left(V_{CB2} \cdot G_M + V_M \cdot G_{CB2\_XY}\right)} = 7.022 \cdot GPa$$

$$G_{CC\_YZ} := \frac{G_{CB1\_YZ} \cdot \left(V_{CB1} \cdot G_{CB2\_YZ} + V_M \cdot G_M\right)}{\left(V_{CB2} + V_M\right) \cdot G_{CB1\_YZ} + V_{CB1} \cdot \left(V_{CB2} \cdot G_{CB2\_YZ} + V_M \cdot G_M\right)} = 10.222 \cdot GPa$$

$$G_{CC\_XZ} := \frac{G_{CB2\_XZ} \cdot \left(V_{CB1} \cdot G_{CB1\_XZ} + V_M \cdot G_M\right)}{\left(V_{CB1} + V_M\right) \cdot G_{CB2\_XZ} + V_{CB2} \cdot \left(V_{CB2} \cdot G_{CB1\_XZ} + V_M \cdot G_M\right)} = 10.717 \cdot GPa$$

$$\nu_{CC\_XY} := V_{CB1} \cdot \nu_{CB1\_XY} + \frac{\nu_M \cdot \nu_{CB2\_XY}}{V_{CB2} \cdot \nu_M + V_M \cdot \nu_{CB2\_XY}} = 0.427$$

$$\nu_{CC\_YZ} := \frac{\nu_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}}{V_{CB1} \cdot \nu_{CB2\_YZ} \cdot \nu_M + V_{CB2} \cdot \nu_{CB1\_YZ} \cdot \nu_M + V_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}} = 0.344$$

$$\nu_{CC\_XZ} := \frac{\nu_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}}{V_{CB1} \cdot \nu_{CB2\_XZ} \cdot \nu_M + V_{CB2} \cdot \nu_{CB1\_XZ} \cdot \nu_M + V_M \cdot \nu_{CB1\_XZ} \cdot \nu_{CB2\_XZ}} = 0.345$$

PF1C Composite Conductor Smeared Mechanical Properties

(Without Cooling Hole)

1.0 Problem Description

To determine the PF1C composite conductor effective monolith/ homogenized/ smeared orthotropic mechanical properties: Elastic Modulus; Shear Modulus; Poisson's Ratio; and Coefficient of Thermal Expansion, using the unit cell method.

Reference: "Effective mechanical properties of EM composite conductors: an analytical and finite element modeling approach", W. Sun, Composite Structures, Volume 58, pg 411-421, 2002

2.0 Given

The details of PF1C composite conductor are shown in Figures 1 thru 4 (note: without the cooling hole).

Metallic Conducting Core Width $L := 19.812 \cdot mm$

Ratio of Core Height-to-Width $a := .782$

Metallic Core Height $aL := a \cdot L = 15.493 \cdot mm$

Metallic Core Young's Modulus $E_M := 110 \cdot GPa$

Metallic Core Poisson's Ratio $\nu_M := .34$

Metallic Core Shear Modulus $G_M := \dfrac{E_M}{2 \cdot (1 + \nu_M)} = 41.045 \cdot GPa$

Phase 1 Insulation Conductor Width-to-Thickness Ratio $b := .0089744$

Phase 1 Insulation Thickness $bL := b \cdot L = 0.178 \cdot mm$

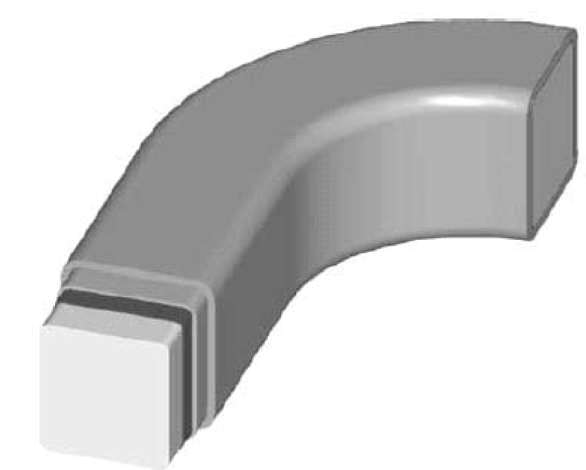


Fig. 1. Schematic view of a conductor structural winding.

Phase 1 Insulation Young's Modulus	$E_{P1} := 2.5 \cdot GPa$
Phase 1 Insulation Poisson's Ratio	$\nu_{P1} := .34$
Phase 1 Insulation Shear Modulus	$G_{P1} := \frac{E_{P1}}{2 \cdot (1 + \nu_{P1})} = 0.933 \cdot GPa$
Phase 2 Insulation Conductor Width-to-Thickness Ratio	$c := .0089744$
Phase 2 Insulation Thickness	$cL := c \cdot L = 0.178 \cdot mm$
Phase 2 Insulation Young's Modulus	$E_{P2} := 2.5 \cdot GPa$
Phase 2 Insulation Poisson's Ratio	$\nu_{P2} := .34$
Phase 2 InsulationShear Modulus	$G_{P2} := \frac{E_{P2}}{2 \cdot (1 + \nu_{P2})} = 0.933 \cdot GPa$
Phase 3 Insulation Conductor Width-to-Thickness Ratio	$d := .034615$
Phase 3 Insulation Thickness	$dL := d \cdot L = 0.686 \cdot mm$
Phase 3 Fabric Weave	Plain
Phase 3 Fiber +/- Angle	$\theta := 22.5 \cdot deg$
Phase 3 Glass Fiber Young's Modulus	$E_f := 87.5 \cdot GPa$
Phase 3 Glass Fiber Poisson's Ratio	$\nu_f := .23$
Phase 3 Glass Fiber Shear Modulus	$G_f := \frac{E_f}{2 \cdot (1 + \nu_f)} = 35.569 \cdot GPa$

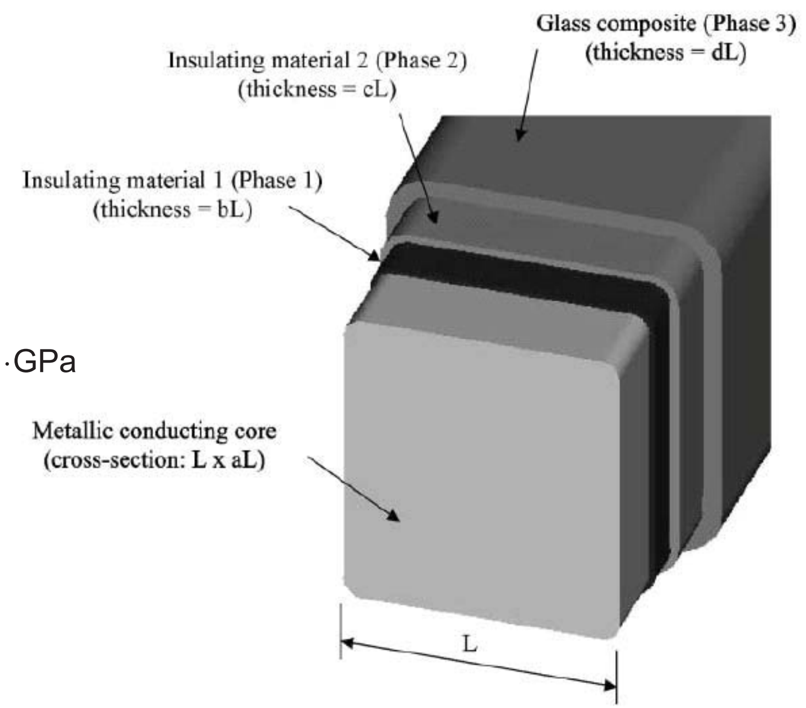


Fig. 2. Schematic representation of a conductor unit cell.

Phase 3 Glass Fiber Volume Fraction $V_f := .5$

Phase 3 Epoxy Young's Modulus $E_m := 4 \cdot GPa$

Phase 3 Epoxy Poisson's Ratio $\nu_m := .36$

Phase 3 Epoxy Shear Modulus $G_m := \dfrac{E_m}{2 \cdot (1 + \nu_m)} = 1.471 \cdot GPa$

Phase 3 Epoxy Volume Fraction $V_m := .5$

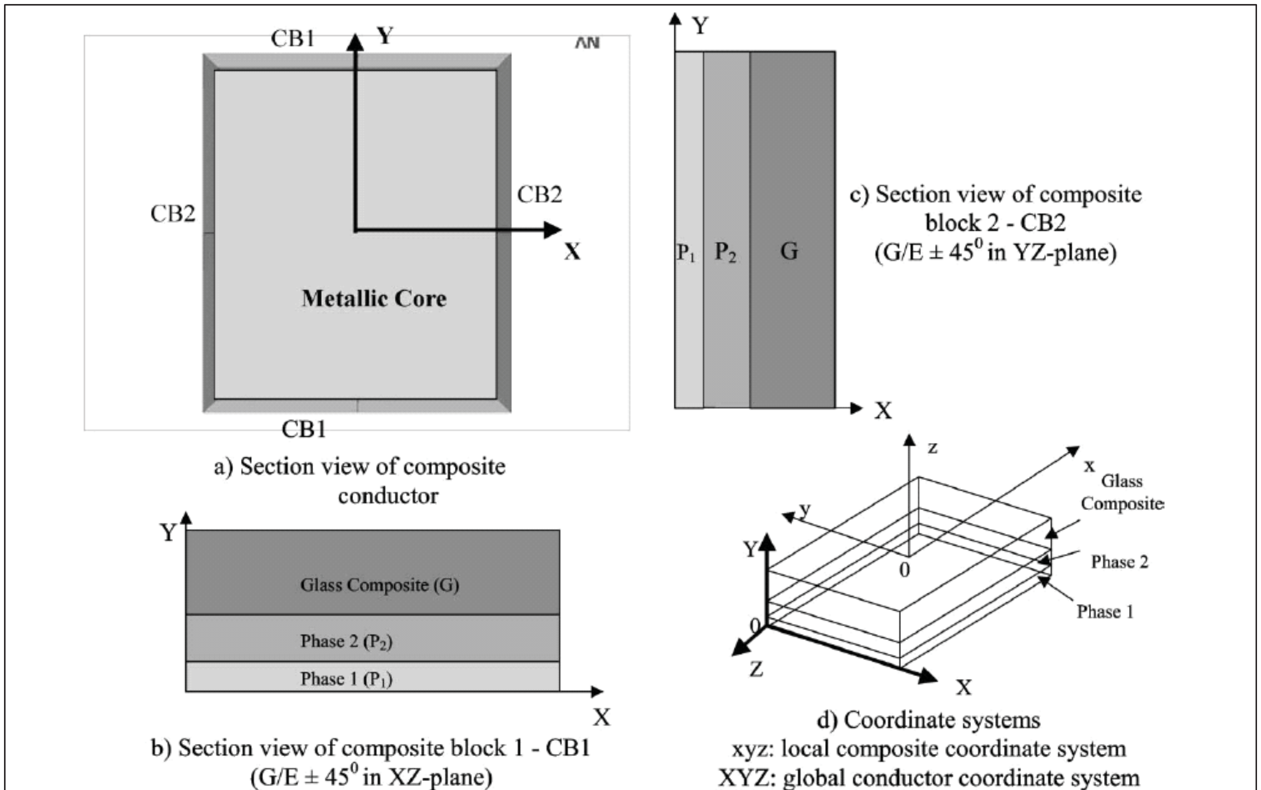


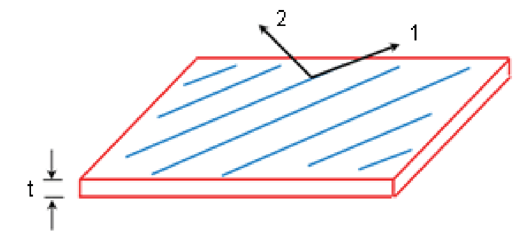
Fig. 4. Section views of conductor and insulation composite blocks. Section view of (a) composite conductor, (b) composite block 1—CB1 ($G/E \pm 45°$ in XZ-plane), (c) composite block 2—CB2 ($G/E \pm 45°$ in YZ-plane), and (d) coordinate systems, xyz: local composite coordinate system; XYZ: global conductor coordinate system.

3.0 Elastic Constants of Woven Glass/ Epoxy Composite Laminae in Local (x,y,z) Coordinate System

3.1 The in-plane elastic constants in the fiber (1, 2, 3) coordinate system

The in-plane Elastic Modulus along the axis (longitudinal direction) of the fiber is given by:

$$E_{11} := V_f \cdot E_f + V_m \cdot E_m = 45.75 \cdot GPa$$



The Poisson's Ratio is given by:

$$\nu_{12} := V_f \cdot \nu_f + V_m \cdot \nu_m = 0.295$$

The Shear Modulus is given by:

$$G_{12} := G_m \cdot \frac{(G_f + G_m) + V_f \cdot (G_f - G_m)}{(G_f + G_m) - V_f \cdot (G_f - G_m)} = 3.979 \cdot GPa$$

The in-plane Elastic Modulus normal to the axis (lateral direction) to the fiber is given by:

$$E_{22} := \frac{\left(V_f + V_m \cdot \frac{E_m}{E_f}\right) \cdot \left[V_f + V_m \cdot .5 \cdot \left(1 + \frac{E_m}{E_f}\right)\right]}{\left(V_f + V_m \cdot \frac{E_m}{E_f}\right) \cdot \left[\frac{V_f}{E_f} + .5 \cdot \left(1 + \frac{E_m}{E_f}\right) \cdot \frac{V_m}{E_m}\right] + V_f \cdot V_m \cdot \left(\frac{-\nu_m}{E_m} + \frac{\nu_f}{E_f}\right) \cdot \frac{\left(\frac{-\nu_f}{E_f} + \frac{\nu_m}{E_m}\right) \cdot \left[\frac{E_m}{E_f} - .5 \cdot \left(1 + \frac{E_m}{E_f}\right)\right]}{\left(\frac{1}{E_f} - \frac{1}{E_m}\right)}} = 11.94 \cdot GPa$$

The Shear Modulus is given by:

$$G_{23} := \frac{.5\cdot\left[V_f + V_m\cdot .5\cdot\left(1+\frac{E_m}{E_f}\right)\right]}{\left[V_f\cdot\left(\frac{1}{E_f}+\frac{\nu_f}{E_f}\right)+V_m\cdot .5\cdot\left(1+\frac{E_m}{E_f}\right)\cdot\left(\frac{1}{E_m}+\frac{\nu_m}{E_m}\right)\right]} = 3.969\cdot GPa$$

The Poisson's Ratio is given by:

$$\nu_{23} := \nu_{12} = 0.295$$

The out-of-plane Elastic Modulus (transverse direction) is given by:

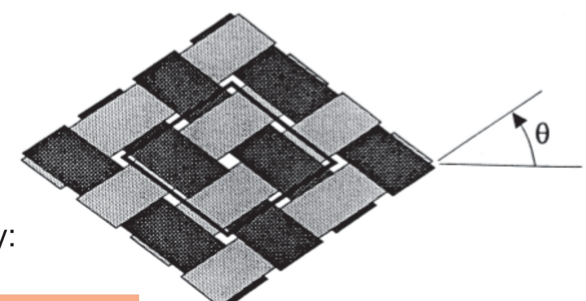
$$E_{33} := \frac{E_f\cdot E_m}{V_f\cdot E_m + V_m\cdot E_f} = 7.65\cdot GPa$$

The Shear Modulus is given by:

$$G_{13} := G_{23} = 3.969\cdot GPa$$

The Poisson's Ratio is given by:

$$\nu_{13} := \nu_{12} = 0.295$$



3.2 The in-plane elastic constants in the local (x, y, z) coordinate system

The in-plane Elastic Modulus of the glass-epoxy laminae in the local x direction is given by:

$$E_{G\_xx} := \frac{E_{11}\cdot E_{22}}{E_{22}\cdot\cos(\theta)^4 + E_{11}\cdot E_{22}\cdot\left(\frac{1}{G_{12}}-\frac{2\cdot\nu_{12}}{E_{11}}\right)\cdot\sin(\theta)^2\cdot\cos(\theta)^2 + E_{11}\cdot\sin(\theta)^4} = 21.042\cdot GPa$$

The in-plane Elastic Modulus of the glass-epoxy laminae in the local y direction is given by:

$$E_{G\_yy} := \frac{E_{11} \cdot E_{22}}{E_{11} \cdot \cos(\theta)^4 + E_{11} \cdot E_{22} \cdot \left(\frac{1}{G_{12}} - \frac{2 \cdot \nu_{12}}{E_{11}} \right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2 + E_{22} \cdot \sin(\theta)^4} = 10.954 \cdot \mathrm{GPa}$$

The Poisson's Ratio is given by:

$$\nu_{G\_xy} := E_{G\_xx} \cdot \left[\frac{\nu_{12}}{E_{11}} \cdot \left(\sin(\theta)^4 + \cos(\theta)^4 \right) - \left(\frac{1}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}} \right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2 \right] = 0.485$$

The Shear Modulus is given by:

$$G_{G\_xy} := \frac{G_{12}}{2 \cdot G_{12} \cdot \left(\frac{2}{E_{11}} + \frac{2}{E_{22}} + \frac{4 \cdot \nu_{12}}{E_{11}} - \frac{1}{G_{12}} \right) \cdot \sin(\theta)^2 \cdot \cos(\theta)^2 + \left(\sin(\theta)^4 + \cos(\theta)^4 \right)} = 5.408 \cdot \mathrm{GPa}$$

The Elastic Modulus in the local z direction is given by:

$$E_{G\_zz} := E_{33} = 7.65 \cdot \mathrm{GPa}$$

And the remaining constants are given by:

$$G_{G\_yz} := G_{23} = 3.969 \cdot \mathrm{GPa}$$

$$G_{G\_xz} := G_{13} = 3.969 \cdot \mathrm{GPa}$$

$$\nu_{G\_yz} := \nu_{23} = 0.295$$

$$\nu_{G\_xz} := \nu_{13} = 0.295$$

4.0 Elastic Constants of Insulation Composite Laminae in the Global (X,Y,Z) Coordinate System

4.1 Elastic constants of insulation composite CB1 laminae in the global (X,Y,Z) coordinate system

The volume fraction each of the 3 phases in the CB1 laminae are given by:

Volume Fraction of Phase 1 $$V_{1\_P1} := \frac{b}{(b+c+d)} = 0.171$$

Volume Fraction of Phase 2 $$V_{1\_P2} := \frac{c}{(b+c+d)} = 0.171$$

Volume Fraction of Phase 3 $$V_{1\_G} := \frac{d}{(b+c+d)} = 0.659$$

The elastic constants for composite block CB1 are given by:

$$E_{CB1\_XX} := V_{1\_P1} \cdot E_{P1} + V_{1\_P2} \cdot E_{P2} + V_{1\_G} \cdot E_{G\_yy} = 8.067 \cdot GPa$$

$$E_{CB1\_YY} := \frac{E_{P1} \cdot E_{P2} \cdot E_{G\_zz}}{V_{1\_P1} \cdot E_{P2} \cdot E_{G\_zz} + V_{1\_P2} \cdot E_{P1} \cdot E_{G\_zz} + V_{1\_G} \cdot E_{P1} \cdot E_{P2}} = 4.491 \cdot GPa$$

$$E_{CB1\_ZZ} := V_{1\_P1} \cdot E_{P1} + V_{1\_P2} \cdot E_{P2} + V_{1\_G} \cdot E_{G\_xx} = 14.711 \cdot GPa$$

$$\nu_{CB1\_XY} := V_{1\_P1} \cdot \nu_{P1} + V_{1\_P2} \cdot \nu_{P2} + V_{1\_G} \cdot \nu_{G\_yz} = 0.31$$

$$G_{CB1\_XY} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_yz}}{V_{1\_P1} \cdot G_{P2} \cdot G_{G\_yz} + V_{1\_P2} \cdot G_{P1} \cdot G_{G\_yz} + V_{1\_G} \cdot G_{P2} \cdot G_{P1}} = 1.88 \cdot GPa$$

$$G_{CB1\_YZ} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_xz}}{V_{1\_P1} \cdot G_{P2} \cdot G_{G\_xz} + V_{1\_P2} \cdot G_{P1} \cdot G_{G\_xz} + V_{1\_G} \cdot G_{P2} \cdot G_{P1}} = 1.88 \cdot GPa$$

$$G_{CB1\_XZ} := V_{1\_P1} \cdot G_{P1} + V_{1\_P2} \cdot G_{P2} + V_{1\_G} \cdot G_{G\_xy} = 3.88 \cdot GPa$$

$$\nu_{CB1\_YZ} := V_{1\_P1} \cdot \nu_{P1} + V_{1\_P2} \cdot \nu_{P2} + V_{1\_G} \cdot \nu_{G\_xz} = 0.31$$

$$\nu_{CB1\_XZ} := \frac{\nu_{P1} \cdot \nu_{P2} \cdot \nu_{G\_xy}}{V_{1\_P1} \cdot \nu_{P2} \cdot \nu_{G\_xy} + V_{1\_P2} \cdot \nu_{P1} \cdot \nu_{G\_xy} + V_{1\_G} \cdot \nu_{P1} \cdot \nu_{P2}} = 0.423$$

4.2 Elastic constants of insulation composite CB2 laminae in the global (X,Y,Z) coordinate system

The volume fraction each of the 3 phases in the CB1 laminae are given by:

Volume Fraction of Phase 1 $$V_{2\_P1} := \frac{b}{(b + c + d)} = 0.171$$

Volume Fraction of Phase 2 $$V_{2\_P2} := \frac{c}{(b + c + d)} = 0.171$$

Volume Fraction of Phase 3 $$V_{2\_G} := \frac{d}{(b + c + d)} = 0.659$$

The elastic constants for composite block CB2 are given by:

$$E_{CB2\_XX} := \frac{E_{P1} \cdot E_{P2} \cdot E_{G\_zz}}{V_{2\_P1} \cdot E_{P2} \cdot E_{G\_zz} + V_{2\_P2} \cdot E_{P1} \cdot E_{G\_zz} + V_{2\_G} \cdot E_{P1} \cdot E_{P2}} = 4.491 \cdot GPa$$

$$E_{CB2\_YY} := V_{2\_P1} \cdot E_{P1} + V_{2\_P2} \cdot E_{P2} + V_{2\_G} \cdot E_{G\_yy} = 8.067 \cdot GPa$$

$$E_{CB2\_ZZ} := V_{2\_P1} \cdot E_{P1} + V_{2\_P2} \cdot E_{P2} + V_{2\_G} \cdot E_{G\_xx} = 14.711 \cdot GPa$$

$$\nu_{CB2\_XY} := V_{2\_P1} \cdot \nu_{P1} + V_{2\_P2} \cdot \nu_{P2} + V_{2\_G} \cdot \nu_{G\_yz} = 0.31$$

$$G_{CB2\_XY} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_yz}}{V_{2\_P1} \cdot G_{P2} \cdot G_{G\_yz} + V_{2\_P2} \cdot G_{P1} \cdot G_{G\_yz} + V_{2\_G} \cdot G_{P1} \cdot G_{P2}} = 1.88 \cdot GPa$$

$$G_{CB2\_YZ} := V_{2\_P1} \cdot G_{P1} + V_{2\_P2} \cdot G_{P2} + V_{2\_G} \cdot G_{G\_xy} = 3.88 \cdot GPa$$

$$G_{CB2\_XZ} := \frac{G_{P1} \cdot G_{P2} \cdot G_{G\_xz}}{V_{2\_P1} \cdot G_{P2} \cdot G_{G\_xz} + V_{2\_P2} \cdot G_{P1} \cdot G_{G\_xz} + V_{2\_G} \cdot G_{P1} \cdot G_{P2}} = 1.88 \cdot GPa$$

$$\nu_{CB2\_YZ} := \frac{\nu_{P1} \cdot \nu_{P2} \cdot \nu_{G\_xy}}{V_{2\_P1} \cdot \nu_{P2} \cdot \nu_{G\_xy} + V_{2\_P2} \cdot \nu_{P1} \cdot \nu_{G\_xy} + V_{2\_G} \cdot \nu_{P1} \cdot \nu_{P2}} = 0.423$$

$$\nu_{CB2\_XZ} := V_{2\_P1} \cdot \nu_{P1} + V_{2\_P2} \cdot \nu_{P2} + V_{2\_G} \cdot \nu_{G\_xz} = 0.31$$

5.0 Elastic Constants of Insulation Composite Conductor in the Global (X,Y,Z) Coordinate System

5.1 Volume fraction of the composite conductor components

The overall width of the composite conductor is given by:

$$L_X := L + 2 \cdot L \cdot (b + c + d) = 21.895 \cdot mm$$

The average width for CB1 is

$$L_{X\_CB1} := L + L \cdot (b + c + d) = 20.853 \cdot mm$$

The overall height of the composite conductor is given by:

$$L_Y := aL + 2 \cdot L \cdot (b + c + d) = 17.576 \cdot mm$$

The average height for CB2 is

$$L_{Y\_CB2} := aL + L \cdot (b + c + d) = 16.534 \cdot mm$$

The total cross sectional area of the composite conductor is given by:

$$A_{ZZ} := L_X \cdot L_Y = 384.818 \cdot mm^2$$

The cross sectional area of the metal core is given by:

$$A_M := L \cdot aL = 306.947 \cdot mm^2$$

The cross sectional area of CB1 is given by:

$$A_{CB1} := 2 \cdot L_{X\_CB1} \cdot L \cdot (b + c + d) = 43.433 \cdot mm^2$$

The cross sectional area of CB2 is given by:

$$A_{CB2} := 2 \cdot L_{Y\_CB2} \cdot L \cdot (b + c + d) = 34.438 \cdot mm^2$$

The volume fraction each of the 3 components of in the composite conductor are given by:

Volume Fraction of Metal Core $\quad V_M := \dfrac{A_M}{A_{ZZ}} = 0.798$

Volume Fraction of CB1 $\quad V_{CB1} := \dfrac{A_{CB1}}{A_{ZZ}} = 0.113$

Volume Fraction of CB2 $\quad V_{CB2} := \dfrac{A_{CB2}}{A_{ZZ}} = 0.089$

5.2 Elastic constants for the composite conductor

The elastic constants for the composite conductor are given by:

$$E_{CC\_XX} := V_{CB1} \cdot E_{CB1\_XX} + \frac{E_M \cdot E_{CB2\_XX}}{V_{CB2} \cdot E_M + V_M \cdot E_{CB2\_XX}} = 37.705 \cdot GPa$$

$$E_{CC\_YY} := V_{CB2} \cdot E_{CB2\_YY} + \frac{E_M \cdot E_{CB1\_YY}}{V_{CB1} \cdot E_M + V_M \cdot E_{CB1\_YY}} = 31.602 \cdot \mathrm{GPa}$$

$$E_{CC\_ZZ} := V_M \cdot E_M + V_{CB1} \cdot E_{CB1\_XX} + V_{CB2} \cdot E_{CB2\_ZZ} = 89.968 \cdot \mathrm{GPa}$$

$$G_{CC\_XY} := \frac{G_{CB1\_XY} \cdot G_{CB2\_XY} \cdot G_M}{G_{CB2\_XY} \cdot G_M \cdot V_{CB1} + \left(V_{CB2} + V_M\right) \cdot G_{CB1\_XY} \cdot \left(V_{CB2} \cdot G_M + V_M \cdot G_{CB2\_XY}\right)} = 8.367 \cdot \mathrm{GPa}$$

$$G_{CC\_YZ} := \frac{G_{CB1\_YZ} \cdot \left(V_{CB1} \cdot G_{CB2\_YZ} + V_M \cdot G_M\right)}{\left(V_{CB2} + V_M\right) \cdot G_{CB1\_YZ} + V_{CB1} \cdot \left(V_{CB2} \cdot G_{CB2\_YZ} + V_M \cdot G_M\right)} = 11.545 \cdot \mathrm{GPa}$$

$$G_{CC\_XZ} := \frac{G_{CB2\_XZ} \cdot \left(V_{CB1} \cdot G_{CB1\_XZ} + V_M \cdot G_M\right)}{\left(V_{CB1} + V_M\right) \cdot G_{CB2\_XZ} + V_{CB2} \cdot \left(V_{CB2} \cdot G_{CB1\_XZ} + V_M \cdot G_M\right)} = 13.348 \cdot \mathrm{GPa}$$

$$\nu_{CC\_XY} := V_{CB1} \cdot \nu_{CB1\_XY} + \frac{\nu_M \cdot \nu_{CB2\_XY}}{V_{CB2} \cdot \nu_M + V_M \cdot \nu_{CB2\_XY}} = 0.415$$

$$\nu_{CC\_YZ} := \frac{\nu_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}}{V_{CB1} \cdot \nu_{CB2\_YZ} \cdot \nu_M + V_{CB2} \cdot \nu_{CB1\_YZ} \cdot \nu_M + V_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}} = 0.342$$

$$\nu_{CC\_XZ} := \frac{\nu_M \cdot \nu_{CB1\_YZ} \cdot \nu_{CB2\_YZ}}{V_{CB1} \cdot \nu_{CB2\_XZ} \cdot \nu_M + V_{CB2} \cdot \nu_{CB1\_XZ} \cdot \nu_M + V_M \cdot \nu_{CB1\_XZ} \cdot \nu_{CB2\_XZ}} = 0.345$$