

**PPPL Calculation Form - No: NSTXU-CALC-133-24 #**

Calculation # NSTXU-CALC-133-24 Revision # 0 WP #, if any 2254  
(ENG-032)

Purpose of Calculation: (Define why the calculation is being performed.)

The purpose of this calculation is to define the conductor fatigue stress limit and determine the fatigue crack growth life for a minimum detectable flaw size of 0.8 mm achievable according to the conductor suppliers. Prior to the NSTX-U recovery project, the stress results from the NSTX-U OH conductor fatigue and fracture mechanics analyses performed by P. Titus and J. Feng were used for the design of PF1A prototype coils. The inner PF coil design has been evolved and design changes have been made since then based on the newly revised physics requirements for the magnet system. This report is to summarize the fatigue and crack growth life calculation for the new design of the inner PF coils but using the copper fatigue stress test data similar to that used in the crack growth analysis for the OH coil.

Codes and versions: (List all codes, if any, used)

ANSYS 18.2

References (List any source of design information including computer program titles and revision levels.)

- [1] NSTXU-CALC-133-09-00 OH Conductor fatigue and fracture mechanics analyses, Titus, November, 2010
- [2] NSTXU-CALC-133-18-00 PF1A upper and lower replacement stress analysis, Titus, January, 2017
- [3] NSTX-U-SPEC-MAG-001-2 Specification for Inner PF coil conductor, M. Kalish, November, 2017
- [4] NSTX-U-RQMT-GRD-001-00 General Requirements Document, S. Gerhardt, December, 2017
- [5] NSTX-CRIT-0001-02 Structural Design Criteria, I. Zatz, January, 2016

Assumptions (Identify all assumptions made as part of this calculation.)

The most commonly used semi-elliptic crack shape is assumed for a 0.8 mm minimum detectable flaw size in the conductor for all six inner PF coils. Linear elastic fracture mechanics and Paris Law is used for the fatigue crack growth calculation. In addition, assume a crack is propagating only along the through-thickness direction of the conductor. Two dimension crack propagation effect and potential crack arrest due to the dynamic stress release and energy release rate during crack propagation is neglected.

Calculation (Calculation is either documented here or attached)

Please see attached main body of this document.

Conclusion (Specify whether or not the purpose of the calculation was accomplished.)

The fatigue requirements on the peak hoop stress or the maximum principal stress in the inner PF coils for 20,000 cycles are derived for conductor in the coil terminals to satisfy the NSTX-U structural design requirement. A higher allowable of conductor fatigue stress of 160 MPa is derived for the PF1 coil lead sections based on the Linear Elastic Fracture Mechanics Calculation fracture based fatigue requirements.

Cognizant Individual (or designee) printed name, signature, and date  
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**I have reviewed this calculation and, to my professional satisfaction, it is properly performed and correct.**

Checker's printed name, signature, and date

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U.S. DEPARTMENT OF  
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Science



**NSTX-U**

## National Spherical Torus eXperiment - Upgrade

# NSTX-U

## Calculation of Inner PF Coil Fatigue and Fracture Mechanics

NSTXU-CALC-133-24 00

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## NSTX-U CALCULATION

## Record of Changes

[illegible]

## NSTX-U Calculation Form

Purpose of Calculation: The inner PF coils for the NSTX-U recovery project is designed to have 20,000 pulse cycles in which conductors for all six coils will experience a fatigue stress during the machine operation when pulsed or during cool down. This calculation is to define the conductor fatigue stress limit and determine the fatigue crack growth life for a minimum detectable flaw size of 0.8 mm achievable according to the conductor suppliers. Prior to the NSTX-U recovery project, the stress results from the NSTX-U OH conductor fatigue and fracture mechanics analyses performed by P. Titus and J. Feng were used for the design of PF1A prototype coils. The inner PF coil design has been evolved and design changes have been made since then based on the newly revised physics requirements for the magnet system. This report is to summarize the fatigue and crack growth life calculation for the new design of the inner PF coils but using the copper fatigue stress test data similar to that used in the crack growth analysis for the OH coil.

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The most commonly used semi-elliptic crack shape is assumed for a 0.8 mm minimum detectable flaw size in the conductor for all six inner PF coils. Linear elastic fracture mechanics and Paris Law is used for the fatigue crack growth calculation. In addition, assume a crack is propagating only along the through-thickness direction of the conductor. Two dimension crack propagation effect and potential crack arrest due to the dynamic stress release and energy release rate during crack propagation is neglected.

Calculation: *(Calculation is either documented here or attached)*

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## 1. Executive Summary

The Inner PF coils for the NSTX-U are installed to provide the poloidal field shaping and better controlling of the plasma in the divertor region. The inner PFs are designed to have 20,000 pulse cycles over the lifetime of the NSTX-U operation as defined in the latest General Requirement Document [4]. According to the NSTX-U structural design criteria [5], a fatigue strength evaluation is required for those NSTX-U structural components with undetectable flaws that are either cycled over 10,000 times during their operational lives or are exposed to cyclic peak stresses exceeding its yield stress. When a fatigue strength evaluation is performed, it shall apply to both the base metal and braze joint regions. As an important part of the coil design validation process, the fatigue strength evaluation includes meeting requirements of either the design Stress-N (S-N) fatigue curve derived from material test data, or the crack growth limitation for the 20,000 cycles.

## 2. Inner PF Coil Design

The coil geometry and conductor dimension in the transient thermal analysis models are taken from the latest Kalish Coil Design Parameter data sheet [3]. To ensure a self-consistent coil alignment with consideration of assembly and positional tolerances of components, the PF-1a conductor width was reduced by 1 mm since inner PF PDR so to increase the Center Stack Casing inner bore size by 8 mm (4 mm on each side), and the cooling hole size for PF-1a is reduced from 0.225" to 0.185" accordingly so to maintain the same width from hole edges to conductor outer edges for the fatigue crack propagation of 1mm size minimum detectable flaws [8]. Table 1 listed the conductor design parameters for the inner PFs.

**Table 1 – Inner PF Conductor Dimension and Fatigue Crack Path Length**

Conductor	PF1A	PF1B	PF1C	PF1A (mm)	PF1B	PF1C	
Width	0.52	0.54	0.78	13.208	13.716	19.812	mm
Height	0.98	0.5	0.61	24.892	12.7	15.494	mm
hole diam	0.225	0.146	0.146	5.715	3.7084	3.7084	mm
				7.493	10.0076	16.1036	mm
crack path length				3.7465	5.0038	8.0518	mm

### 3. Fatigue Design Limits

According to [5], fatigue S-N (stress vs cycles) fatigue curves shall be obtained based on the uniaxial strain cycling tests at service temperatures and at various R (stress) ratios. S-N fatigue curves shall be developed for both the base metal and for braze joints in the coil lead region.

- The conductor static stress design limit is derived from the minimum yield strength given in the specifications for the inner PF conductors
- The fatigue limit for copper is derived from the copper fatigue S-N curve

Figure 1 presents the copper conductor fatigue S-N curve from test data available from a number of references [1]. For S-N fatigue evaluation, the more strict criteria of 2 on stress and 20 on life must be met. For the fracture mechanics evaluation, a factor of 2 on the minimum detectable flaw size, 1.5 on the fracture toughness, and 2 on life must be met. The measured NSTX OH conductor braze joint fatigue life is also included in the evaluation, along with the published S-N data for comparison.

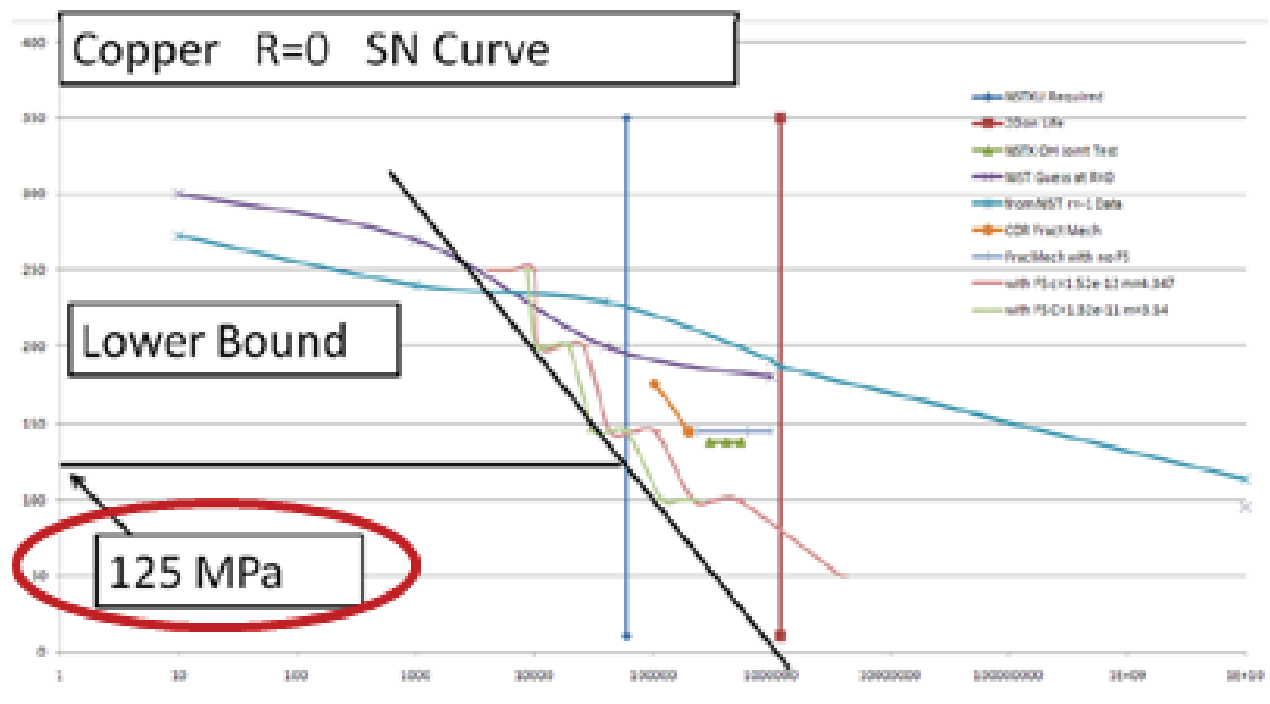


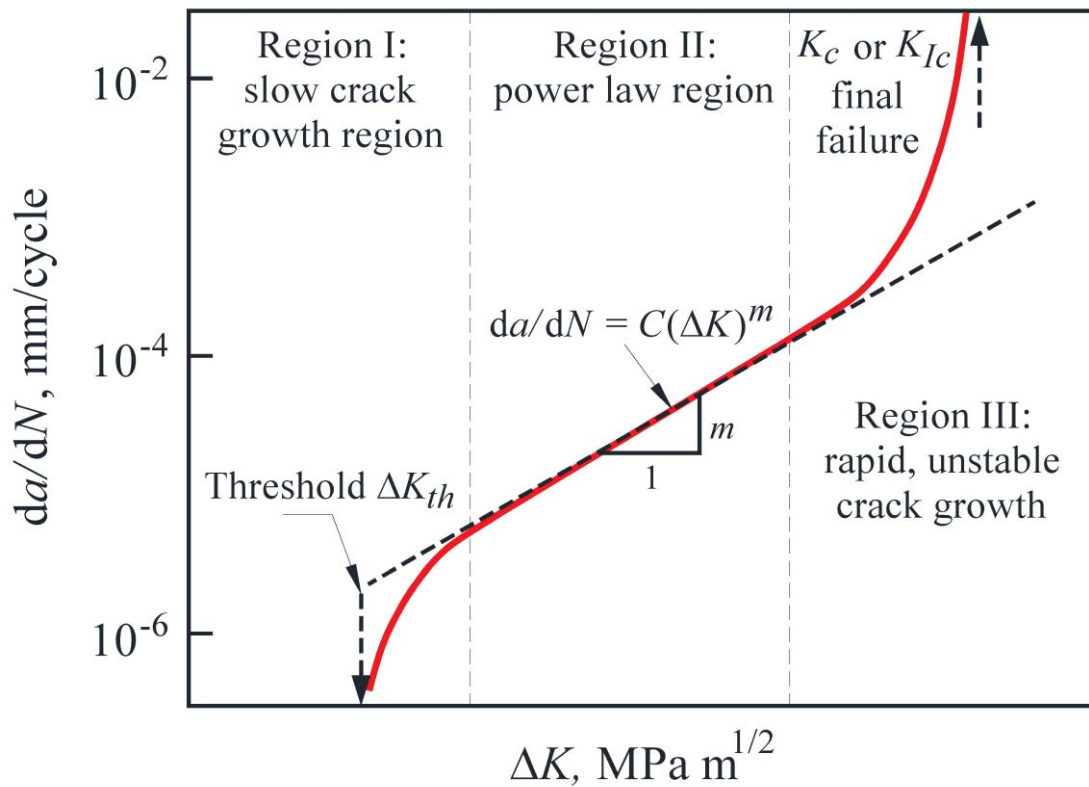
Figure 2 SN and Fracture Mechanics Fatigue Life

Figure 1 Fatigue S-N curve based on test data from a number of references



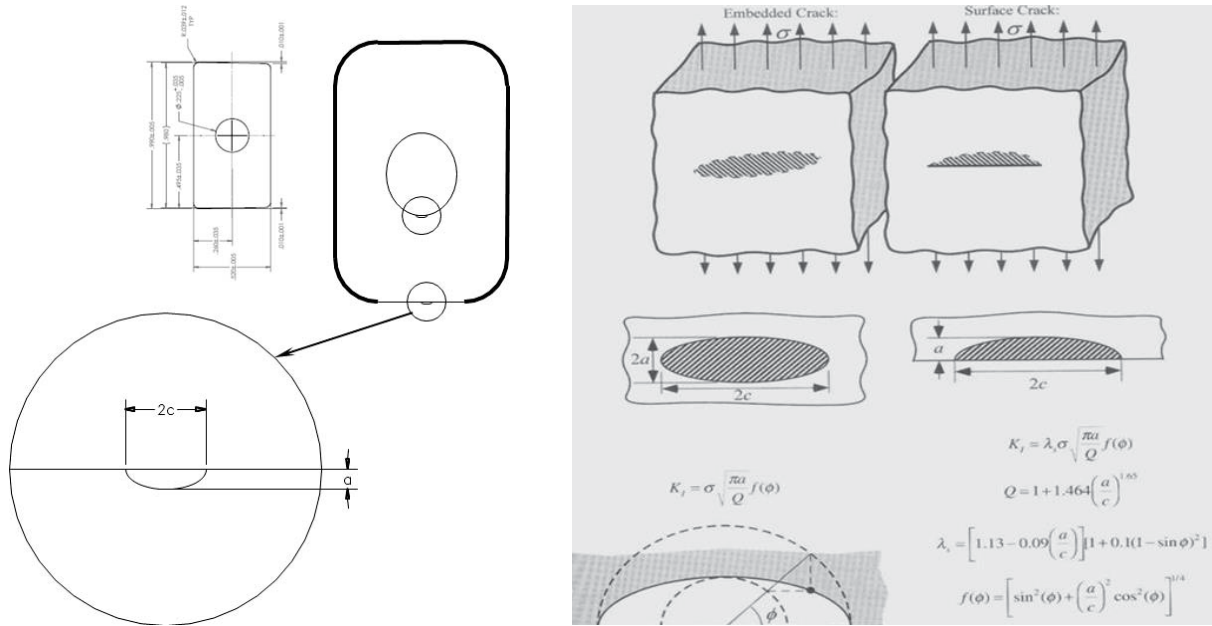
#### 4. Approach for Crack Propagation Analysis

A permissible flaw size is determined from a Linear Elastic Fracture Mechanics (LEFM) analysis for opening mode I crack. The typical fatigue crack growth behavior in metals is shown in Figure 2. The fracture mechanics calculation is the basis of the qualification of the copper conductors used in the inner PF coils. Region 1 in Figure 2 is the crack initiation and a slow crack growth region where crack threshold level is the dominating factor. In region II – the power law region, crack growth rate is linearly proportional to the stress concentration at the crack tip on the log-log scale. Fast fracture is capture in region III where rapid, unstable crack growth should be prevented for brittle materials such as tiles for the plasma facing components.



**Figure 2 Typical fatigue crack growth behavior in metals**

A permissible flaw size is determined from Linear Elastic Fracture Mechanics analysis for Mode I (opening mode) crack. A single, semi-elliptic crack is assumed with a crack depth to width ratio of  $a/c = 1/3$ , where  $a$  represents the crack size of minor axis (depth) and  $c$  is the crack size of the major axis (width). Figure 3 presents a typical semi-elliptic surface crack for the Mode I (opening mode) crack growth behavior.



**Figure 3 A semi-elliptic Surface Crack for Mode I Fatigue Crack Growth**

For hardened copper such as that used in the inner PF coils, the following material fatigue properties from J. Feng at MIT are used [1]

OFHC copper (C10200) Paris constant:  $C = 1.32 \times 10^{-11}$  m/cycle,  $m = 3.54$ ;  
 Fracture toughness:  $K_{1C} = 150 \text{ MPam}^{1/2}$   
 Walker's coefficient: 0.8

Hardened copper Paris parameter:  $C = 1.52 \times 10^{-12}$  m/cycle,  $m = 4.347$ ;  
 Fracture toughness:  $K_{1C} = 150 \text{ MPam}^{1/2}$   
 Walker's coefficient: 0.8

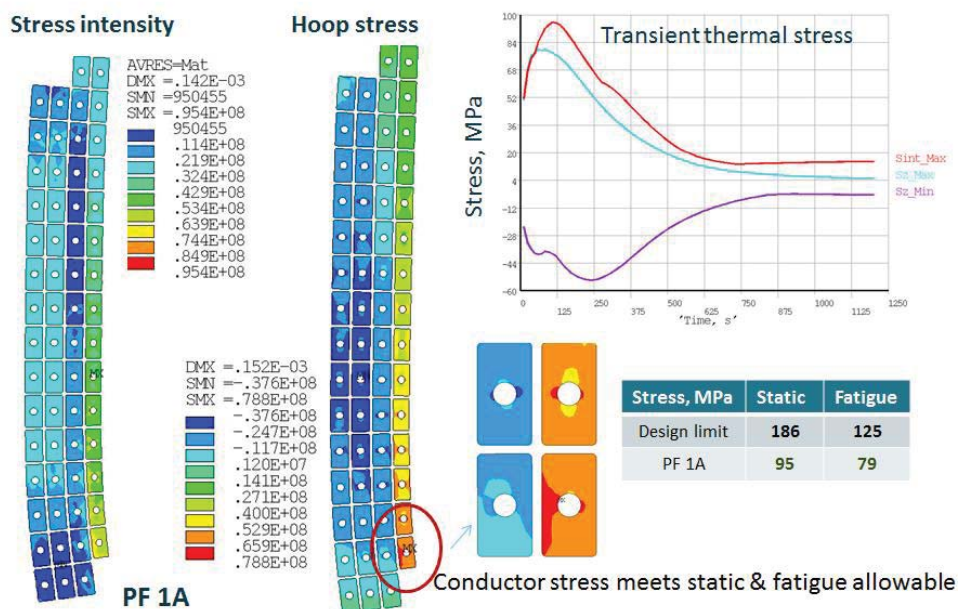
Another set of material properties for base metal used by P. Titus in the OH coil fatigue crack analysis include [1], Paris parameter:  $C = 1.18 \times 10^{-11}$  m/cycle,  $m = 4.55$ . Since the resultant number of cycles for crack propagation is very sensitive to the material properties used for base metal. A better list of references for the three sets of reference data will be useful.

## 5. Fatigue Crack Growth

The inner PFs will experience both EM and thermal fatigue stress in the conductor but thermal stress during cool down of coils dominates the fatigue evaluation for the coil winding pack and EM stress due to Lorentz loads dominates fatigue evaluation for coil leads. During normal operations, plasma currents are ramped in a second up to 2 MA and on for 5 s flat top and then ramped down in 2 seconds. The equivalent square wave time of PF1 a/b/c coils is 1.9, 1.0 and 1.4 seconds respectively. The maximum thermal

stresses in the PF winding packs are shown in Figures 4-6 for PF-1a, PF-1b and PF-1c respectively. Tresca stress or stress intensity is used for the static evaluation and the maximum hoop stress is used for fatigue crack growth evaluation if the fatigue S-N curve cannot be satisfied. Figures 4-6 below also showed the transient thermal stresses including stress intensity and maximum hoop stress distribution in the inner PFs during cool down phase.

## Transient Thermal, Conductor Stress



**Figure 4 PF-1a Conductor Stress during Transient Cool Down**

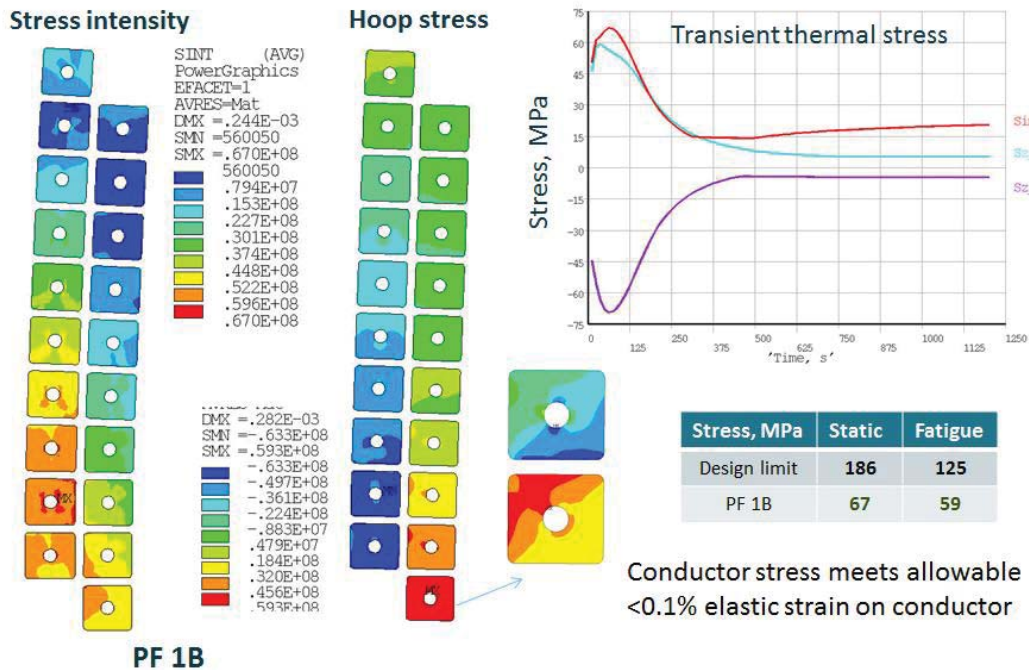


Figure 5 PF-1b Conductor Stresses during Transient Cool Down

## Transient Thermal, Conductor Stress

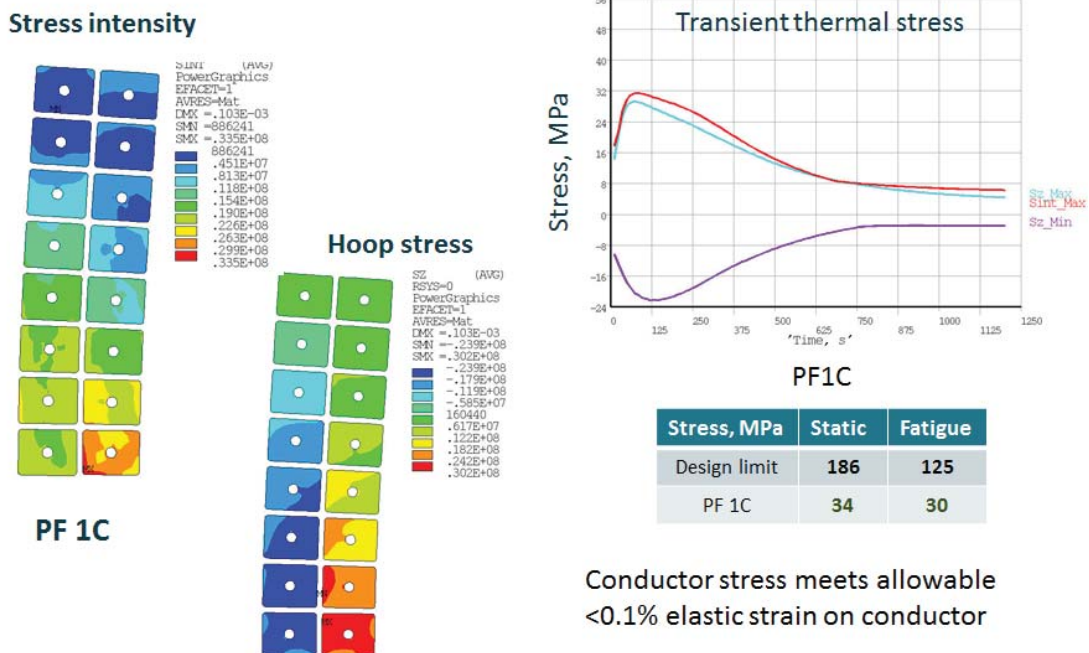


Figure 6 PF-1c Conductor Stresses during Transient Cool Down

For the lead section of the conductor at coil terminals, local higher peak stresses around braze joint interfaces with the bus flags can be found, although this local high peak stress is mainly due to geometry discontinuity, a fatigue crack growth evaluation is performed to fully qualify conductors in the coil leads. Local structural reinforcement with new design of the filler blocks, support brackets, and bus bar flags will be applied whenever possible to finalize the interface design with the coil terminals. For now the crack growth evaluation is based on the maximum peak stresses in the conductor of lead sections for all six coils and will be presented at the FDR.

Table 2 presents the maximum conductor stresses obtained from the detailed 3D coil lead and bus bar analysis. Since the peak stresses are higher than the 125 MPa defined for the OH conductors. A separate fatigue crack growth calculation is then performed using the maximum detectable flaw size defined by the vender (0.8 mm). Figure 7 presents the modified conductor size for the PF-1a coils where the 3.7465 mm crack growth path is preserved. Table 3 presents details of the fatigue crack growth calculation for initial maximum stresses defined by Table 2 from coil lead and bus bar analysis. Figure 8 presented the stress vs. number of cycles from Paris Law. The results show that 160 MPa can be tolerated for a 0.8 mm crack to propagate through the 3.75 mm thick conductor without leaking.

**Table 2 – Maximum Conductor Stresses in Coil Leads – End of Pulse**

<b>Stress (MPa)</b>	<b>PF-1a</b>	<b>PF-1b</b>	<b>PF-1c</b>
<b>EQ #</b>	51	33	18
<b>Peak Stress (positive TF)</b>	160	124	130
<b>Peak Stress (reverse TF)</b>	159	175	120
<b>Fatigue Allowable</b>	160	>180	>200

It should be noted that to be conservative the calculations presented in Table 3 and Figure 8 used only the Paris Law in Linear Elastic Fracture Mechanics and the stress intensity factor, but not the J stress contour integral, which can take into account the plastic deformation in crack tips. This will further increase the number of cycles achieved.



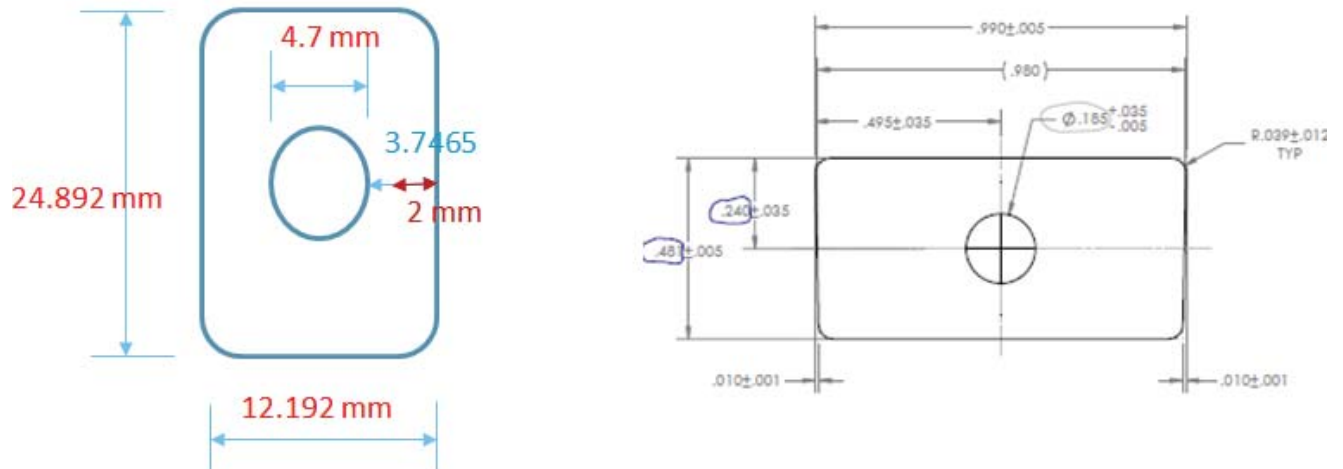


Figure 7 Modified Conductor Size for PF-1a with Crack Path Length Preserved

Table 3 – Fatigue Crack Growth Calculation with Maximum Stresses in Coil Leads

Fracture Mechanics, Fundamentals and Applications, Second Edition, T.L. Anderson

Inner PF coils

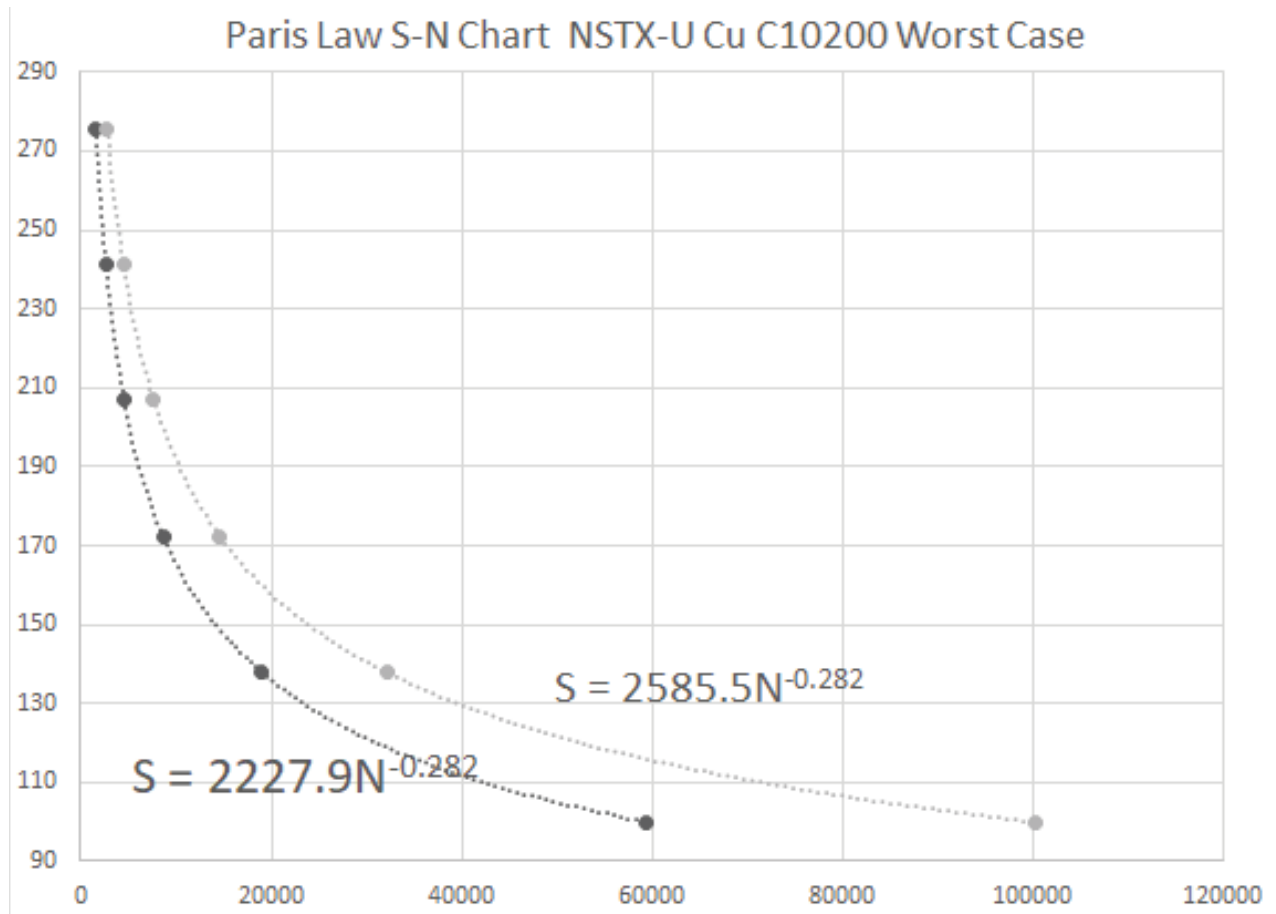
Conduit Geometry

t = 2 mm c = 1.32E-11  
W = 24.892 mm m = 3.54  
σ = 125 MPa

total No. cycle divided by 4  
980,938 245,234

a/c = 0.333  
Δa = 0.01

a <sub>i</sub>	a <sub>i+1</sub>	a <sub>avg</sub>	c	φ	Q	M1	M2	M3	f <sub>i</sub>	f <sub>e</sub>	g	F	ΔK	ΔN	total cycles	factor of 4
1.6	1.61	1.605	4.81982	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0190	1.0037	1.5709	12.53	98358.1		24589.53
1.61	1.62	1.615	4.84985	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0193	1.0037	1.5751	12.60	96362.7		
1.62	1.63	1.625	4.87988	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0197	1.0037	1.5793	12.67	94423.4		
1.63	1.64	1.635	4.90991	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0201	1.0038	1.5835	12.75	92538.7		
1.64	1.65	1.645	4.93994	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0205	1.0038	1.5876	12.82	90707.0		
1.65	1.66	1.655	4.96997	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0208	1.0038	1.5917	12.89	88926.6		
1.66	1.67	1.665	5	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0212	1.0038	1.5957	12.96	87196.0		
1.67	1.68	1.675	5.03003	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0216	1.0039	1.5997	13.03	85513.8		
1.68	1.69	1.685	5.06006	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0220	1.0039	1.6037	13.11	83878.6		
1.69	1.7	1.695	5.09009	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0224	1.0039	1.6076	13.18	82289.1		
1.7	1.71	1.705	5.12012	90	1.238547	1.10003	1.129794	-0.51645	0.9520	1.0228	1.0040	1.6115	13.25	80743.8	980937.8	245234.5



**Figure 8 Stress vs. Cycles for PF1 conductors with Crack Growth Length of 2 mm (black) and 3.75 mm (gray). The modified 1a conductor maintained 3.75 mm crack growth path.**

Conclusion: *(Specify whether or not the purpose of the calculation was accomplished)*

The fatigue requirements on the peak hoop stress or the maximum principal stress in the inner PF coils for 20,000 cycles are derived for conductor in the coil terminals to satisfy the NSTX-U structural design requirement. A higher allowable of conductor fatigue stress of 160 MPa is derived for the PF1 coil lead sections based on the Linear Elastic Fracture Mechanics Calculation fracture based fatigue requirements. A limit of 125 MPa is chosen for the coil windings

