NSTX U

Torques on TF Conductors & Resulting Torsion & Shear Stress in NSTX CSU

NSTX U CALC 132-03-01
Rev.1

March 12, 2015

Prepared By:

Robert D. Woolley
Digitally signed by Robert D. Woolley
DN: cn=Robert D. Woolley, o=Princeton Plasma Physics Laboratory, ou, email=woolley@pppl.gov, c=US
Date: 2015.03.12 14:51:51 -04'00'

R. Woolley

Reviewed By:

Peter H. Titus
Digitally signed by Peter H. Titus
DN: cn=Peter H. Titus, o, ou=Princeton Plasma Physics Laboratory, email=ptitus@pppl.gov, c=US
Date: 2015.03.13 07:55:25 -04'00'

P. Titus, PPPL Engineering Analysis Branch

Reviewed By

Timothy N. Stevenson
Digitally signed by Timothy N. Stevenson
DN: cn=Timothy N. Stevenson, o=PPPL, ou=ENGR, email=tstevenson@pppl.gov, c=US
Date: 2015.03.13 15:38:15 -05'00'

T. Stevenson, Cognizant Engineer

REVISION SHEET

Document No. 132-03-01 **Rev. 1 (Previous rev. was 132-03-00)**

Description	Prepared by	Date
(1) TF torque coefficient sets were determined and included for the TF upper outer leg and for the TF lower outer leg. (2) **TF torque coefficients from OH current as calculated for the 04-May-2010 design point and included in the previous revision of this calculation were based on more OH turns than were included in the final OH coil design and were therefore erroneous. The torque coefficients' values are corrected herein.**	**R. Woolley**	**12March'15**

PPPL Calculation Form

Calculation # **132-03-01** Revision **#01** WP #, if any (ENG-032) **0029, 0037**

Purpose of Calculation: (Define why the calculation is being performed.)

The purpose of this calculation is to provide:

1. **A means of calculating the torsional moments acting on the TF system for use in the Design Point Spreadsheet. These moments were used to size many of the OOP support structures and/or to check Lorentz force calculations.**
2. **Calculations of the Inner Leg Torsional Shear Stress and an appropriate algorithm for the DCPS to compute the torsional shear stress to qualify the TF and/or to qualify similar results in ref.5.**

References (List any source of design information including computer program titles and revision levels.)

1. **R. Woolley memo 13-260709 "Out-of-Plane (OOP) PF/TF Torques on TF Conductors in NSTX CSU" 26 July 2009**
2. **P. Titus Memo, "Maximum TF Torsional Shear", 29 July 2009**
3. **P.Titus paper, "Properties for Out of Plane Support of the TF coils in recent tokamaks", 28September 1999**
4. **R.Woolley memo, 13-110211, Torques on TF Conductors and Fesulting Torsion & Shear Stress in NSTX-CSU, 04May2010 Design Point", 11 February 2011**
5. **P. Titus, NSTXU-CALC-132-07-00 Rev.0 "TF Inner Leg Torsional Shear Including Input to the DCP{S", 9September 2011**
6. **R.Woolley memo, "Formulae for Out-of-plane on TF condutcors in the NSTX Upgrade", 13 February 2015**

Assumptions (Identify all assumptions made as part of this calculation.)

Stiffnesses for segments of the global structure have come from other FEA calculations. These are judged to be correct, and in so far as they produce torsional shear stresses similar to [5] the stresses used are judged adequate.

Calculation (Calculation is either documented here or attached)

Two memos are included in the body of this calculation document. A 13 February 2015 memo [Ref.6] which includes corrected torsional moment summations that have been extended to included torque sums for the upper outer leg and for the lower outer leg, replaces the 11 February 2011 memo [Ref.4] which formerly had been included in the previous version of this calculation document, while the 18 September 2011 memo [Ref.5] of the previous version is retained without any change.

Conclusion (Specify whether or not the purpose of the calculation was accomplished.)

Final DCPS torque and torsion algorithm coefficients can be obtained from this calculation document.

Cognizant Engineer’s printed name, signature, and date

Timothy N. Stevenson Digitally signed by Timothy N. Stevenson
DN: cn=Timothy N. Stevenson, o=PPPL, ou=ENGR, email=tstevenson@pppl.gov, c=US
Date: 2015.03.13 15:38:51 -05'00'

I have reviewed this calculation and, to my professional satisfaction, it is properly performed and correct.

Checker’s printed name, signature, and date

Peter H. Titus Digitally signed by Peter H. Titus
DN: cn=Peter H. Titus, o, ou=Princeton Plasma Physics Laboratory,
email=ptitus@pppl.gov, c=US
Date: 2015.03.13 07:55:56 -04'00'

To Peter Titus

Date 13 February 2015

From R. Woolley

Subject: Formulae for Out-of-Plane Torques on TF Conductors in the NSTX-Upgrade

References:
1. R. Woolley Memo , "Torques on TF Conductors & Resulting Torsion & Shear Stress in NSTX-CSU, 04May2010 DesignPoint". 11 February 2011
2. R. Woolley Memo, "TF Inner Leg Shear Stress in NSTX-CSU", 13-180911, 18 September 2011
3. NSTXU CALC 132-03-00 rev 0; TORQUES ON TF CONDUCTORS & RESULTING TORSION & SHEAR STRESS IN NSTX_CSU, September 19, 2011

I have calculated TF out-of-plane torque coefficients for the outer-upper portion of outer legs and for the outer-lower portion of the outer legs. These new coefficients, which are in response to your recent request, were not included in Ref.1. I also recalculated torque coefficients which were in Ref.1 for the entire TF outer leg and for the TF upper half. These new recalculated results agreed within <1% with the Ref.1 results for all coils **except for OH**. The new OH coefficient was substantially smaller than in Ref.1, a discrepancy that I subsequently traced to the old coefficient having been calculated for an earlier OH coil design which was planned to have many more turns. The new OH coefficient values here are based on the ***as-built OH*** coil's 880 turns instead of its finally planned 884 turns.

Ref.2 did not present a new algorithm for calculating torques, although its shear stress calculations were based on torques consistent with the 884-turn OH design. Ref.3 included Refs. 1 and 2 without any additional calculations.

The MATLAB algorithms, their data, their results, and the comparison checks I performed on the present calculations are all documented in this memo's included Appendix which is a MATLAB/MSWORD notebook. The new torque algorithms that result are repeated below.

$$\left[\frac{\text{Net TF System OuterLeg Torque}}{1\,\text{N-m}}\right]=\left[\frac{I_{TF}}{130\,\text{kA}}\right]\begin{pmatrix} 3518.8\left[\dfrac{I_{PF1AU}-I_{PF1AL}}{1\,\text{kA}}\right] \\ +3691.4\left[\dfrac{I_{PF1BU}-I_{PF1BL}}{1\,\text{kA}}\right] \\ +4293.4\left[\dfrac{I_{PF1CU}-I_{PF1CL}}{1\,\text{kA}}\right] \\ +13190.4\left[\dfrac{I_{PF2U}-I_{PF2L}}{1\,\text{kA}}\right] \\ +16497.4\left[\dfrac{I_{PF3U}-I_{PF3L}}{1\,\text{kA}}\right] \end{pmatrix}$$

$$\left[\frac{\text{Net Upper Half TF System Torque}}{1\,\text{N-m}}\right] = -\left[\frac{I_{TF}}{130\,\text{kA}}\right]\left(\begin{aligned}&11491.3\left[\frac{I_{OH}}{1\,\text{kA}}\right]+2260.2\left[\frac{I_{PF1AU}+I_{PF1AL}}{1\,\text{kA}}\right]+1580.3\left[\frac{I_{PF1BU}+I_{PF1BL}}{1\,\text{kA}}\right]\\&+1851.3\left[\frac{I_{PF1CU}+I_{PF1CL}}{1\,\text{kA}}\right]+5197.7\left[\frac{I_{PF2U}+I_{PF2L}}{1\,\text{kA}}\right]\\&+22045.8\left[\frac{I_{PF3U}+I_{PF3L}}{1\,\text{kA}}\right]+57019.5\left[\frac{I_{PF4}}{1\,\text{kA}}\right]\\&+118638.0\left[\frac{I_{PF5}}{1\,\text{kA}}\right]+705501\left[\frac{I_{PLASMA}}{1\,\text{MA}}\right]\end{aligned}\right)$$

$$\left[\frac{\text{Net Upper Outer TF System Torque}}{1\,\text{N-m}}\right] = \left[\frac{I_{TF}}{130\,\text{kA}}\right]\left(\begin{aligned}&+1351.9\left[\frac{I_{PF1AU}}{1\,\text{kA}}\right]+2181.2\left[\frac{I_{PF1BU}}{1\,\text{kA}}\right]\\&+2525.1\left[\frac{I_{PF1CU}}{1\,\text{kA}}\right]+8232.2\left[\frac{I_{PF2U}}{1\,\text{kA}}\right]\\&-4528.3\left[\frac{I_{PF3U}}{1\,\text{kA}}\right]-50564.\left[\frac{I_{PF4}}{1\,\text{kA}}\right]\\&-109651.4\left[\frac{I_{PF5}}{1\,\text{kA}}\right]-21026.7\left[\frac{I_{PF3L}}{1\,\text{kA}}\right]\\&-4958.2\left[\frac{I_{PF2L}}{1\,\text{kA}}\right]-1768.3\left[\frac{I_{PF1CL}}{1\,\text{kA}}\right]\\&-1510.2\left[\frac{I_{PF1BL}}{1\,\text{kA}}\right]-2166.9\left[\frac{I_{PF1AL}}{1\,\text{kA}}\right]\\&-12462.6\left[\frac{I_{OH}}{1\,\text{kA}}\right]-634845.5\left[\frac{I_{PLASMA}}{1\,\text{MA}}\right]\end{aligned}\right)$$

$$\left[\frac{\text{Net Lower Outer TF System Torque}}{1\,\text{N-m}}\right] = \left[\frac{I_{TF}}{130\,\text{kA}}\right]\begin{pmatrix} +2166.9\left[\frac{I_{PF1AU}}{1\,\text{kA}}\right]+1510.2\left[\frac{I_{PF1BU}}{1\,\text{kA}}\right] \\ +1768.3\left[\frac{I_{PF1CU}}{1\,\text{kA}}\right]+4958.2\left[\frac{I_{PF2U}}{1\,\text{kA}}\right] \\ 21025.7\left[\frac{I_{PF3U}}{1\,\text{kA}}\right]+50564.\left[\frac{I_{PF4}}{1\,\text{kA}}\right] \\ +109651.4\left[\frac{I_{PF5}}{1\,\text{kA}}\right]+4528.3\left[\frac{I_{PF3L}}{1\,\text{kA}}\right] \\ -8232.2\left[\frac{I_{PF2L}}{1\,\text{kA}}\right]-2525.1\left[\frac{I_{PF1CL}}{1\,\text{kA}}\right] \\ -2181.2\left[\frac{I_{PF1BL}}{1\,\text{kA}}\right]-1351.9\left[\frac{I_{PF1AL}}{1\,\text{kA}}\right] \\ +12462.6\left[\frac{I_{OH}}{1\,\text{kA}}\right]+634845.5\left[\frac{I_{PLASMA}}{1\,\text{MA}}\right] \end{pmatrix}$$

APPENDIX- MATLAB calculation of NSTX-U TF Torque Coefficients and self-checks 13February 2015

New purpose: Get coefs for TF torque formula for upper outer and lower outer, and also recalculate and check old torque coefficients for outer and for upper half.

The same torque methodology is used as was described in my 26July2009 electromagnetic torque model, which determines torques based on differences in PF flux between different locations in the poloidally projected image of the TF coil system.

The (R,Z) points describing the TF coil system were calculated previously from Peter Titus' point set, resulting in the present arrays which include 2000 * 5 equally spaced points on 5 contours as rC and zC, each 2000x5 matrices. The ITF variable is the max TF current (130 kA) times the 36 TF turns. The PF system is described as 24 "windings" of rectangular cross section (including the plasma), each described by its (r,z) center, its dr width and its dz height, and by mn its number of turns. First, load coil system data saved long ago on a thumbdrive memstick, herein called H. Then examine that data and make any needed corrections, then finally do the torque calculations which use a previously documented method based on poloidal magnetic flux..

```
clear
clf
load 'H:\Documents\NSTXCSUGstuff\TorsionModel\torsion.mat'
plot(rC,zC), axis equal
whos
```

```
  Name                                   Size                    Bytes
Class     Attributes

  A                                      7x7                       392
double
  Area                                   1x1                         8
double
  Area95                                 1x1                         8
double
  Area97                                 1x1                         8
double
  Area98                                 1x1                         8
double
  Area99                                 1x1                         8
double
  AreaList                              96x1                       768
double
  BR                                   121x51x32               1579776
double
  BRcoil                               121x51x14                691152
double
  BZ                                   121x51x32               1579776
double
  BZcoil                               121x51x14                691152
double
  C                                   2000x1                     16000
double
```

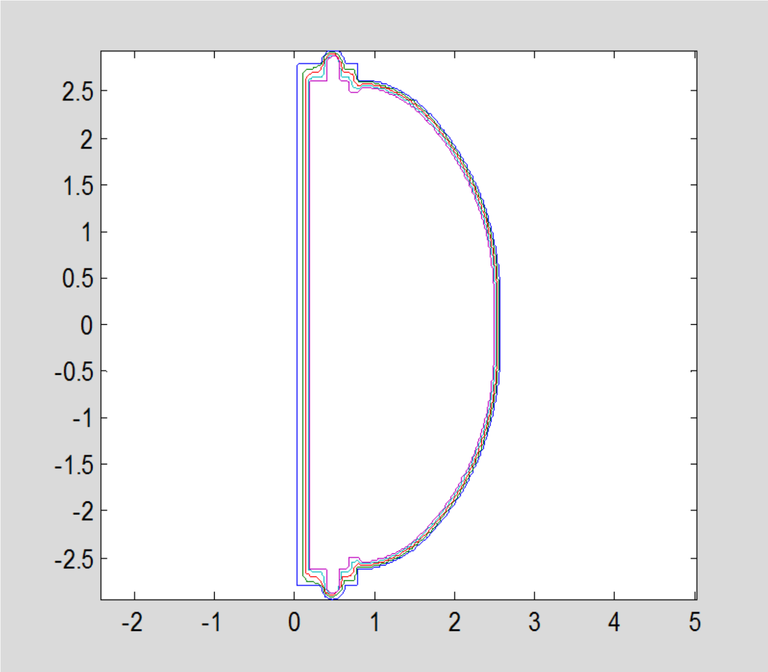
C1	2x1	16	double
C_LCFS	96x1	768	double
Cm	2x474	7584	double
F	7x8	448	double
FLUX	121x51x32	1579776	double
FLUXcoil	121x51x14	691152	double
FLX	121x51x96	4739328	double
FLXinside	96x1	768	double
H	1x24	192	double
Hm	1x1	8	double
I	8x1	64	double
IFIND	858x1	6864	double
II	800x1	6400	double
IND	858x1	6864	double
IND1	858x1	6864	double
ITF	1x1	8	double
I_equilibria	14x96	10752	double
Imax	14x1	112	double
Imin	14x1	112	double
J	8x1	64	double
JFIND	858x1	6864	double
JJ	800x1	6400	double
M_GRID	1x1	8	double
MaxShear	1x1	8	double
MaxShearCases	96x1	768	double
N_GRID	1x1	8	double
RCS_elt	1x1	8	double
RCSe	1x1	8	double

RES	1x1	8	double
RHO	121x51	49368	double
RLe	1x1	8	double
RLeads_elt	1x1	8	double
RMAX	1x1	8	double
RMIN	1x1	8	double
RTFOL_elt	1x1	8	double
RTFOLe	1x1	8	double
R_GRID	1x51	408	double
ShearStress	765x1	6120	double
TorqueState	765x1	6120	double
V	7x7	392	double
XmaxList	96x1	768	double
XminList	96x1	768	double
ZETA	121x51	49368	double
ZMAX	1x1	8	double
ZMIN	1x1	8	double
Z_GRID	1x121	968	double
Zlocation_m	1x1	8	double
ans	32x1	256	double
br	449x249	894408	double
brw	2000x5	80000	double
bz	449x249	894408	double
bzw	2000x5	80000	double
coil_name	24x1	2808	cell
coilcktname	1x14	1688	cell
coilname	14x1	1688	cell
cum_torsion_per_amp2	2000x14	224000	double

```
  flux                                2000x5x14          1120000
double
  flux_from_windings                  2000x5x24          1920000
double
  i                                      1x1                   8
double
  il                                     1x1                   8
double
  iwinding                               1x1                   8
double
  j                                      1x1                   8
double
  jc                                     1x1                   8
double
  k                                      1x1                   8
double
  local_torsion_density_per_amp2      2000x14             224000
double
  mC                                     1x1                   8
double
  mean_flux                           2000x14             224000
double
  model_currents                         7x14                784
double
  mu                                     1x1                   8
double
  mxindex                                1x1                   8
double
  mxshear                                1x1                   8
double
  mxshear_MPa                            1x1                   8
double
  mxtorque                               1x1                   8
double
  mxtorque_Nm                            1x1                   8
double
  nC                                     1x1                   8
double
  nc                                     1x1                   8
double
  num                                   96x13               9984
double
  r1                                     1x1                   8
double
  r10                                    1x1                   8
double
  r11                                    1x1                   8
double
  r12                                    1x1                   8
double
  r13                                    1x1                   8
double
  r14                                    1x1                   8
double
  r15                                    1x1                   8
double
```

r2	1x1	8	double
r3	1x1	8	double
r4	1x1	8	double
r5	1x1	8	double
r6	1x1	8	double
r7	1x1	8	double
r8	1x1	8	double
r9	1x1	8	double
rC	2000x5	80000	double
rf	448x248	888832	double
rhowinding	449x249	894408	double
rwinding	448x248	888832	double
rwindingmax	1x1	8	double
rwindingmin	1x1	8	double
tiling	24x2	384	double
txt	0x0	0	cell
winding_dr	24x1	192	double
winding_dz	24x1	192	double
winding_m	24x1	192	double
winding_mn	24x1	192	double
winding_n	24x1	192	double
winding_name	24x1	2920	cell
winding_r	24x1	192	double
winding_z	24x1	192	double
x	8x1	64	double
xinside	1x1	8	double
xmax	1x1	8	double
xmax95	1x1	8	double

xmax97	1x1	8
double		
xmax98	1x1	8
double		
xmax99	1x1	8
double		
xmin	1x1	8
double		
xmin95	1x1	8
double		
xmin97	1x1	8
double		
xmin98	1x1	8
double		
xmin99	1x1	8
double		
y	8x1	64
double		
yinside	1x1	8
double		
zC	2000x5	80000
double		
zees	765x1	6120
double		
zetawinding	449x249	894408
double		
zf	448x248	888832
double		
zlocation	1x1	8
double		
zwinding	448x248	888832
double		
zwindingmax	1x1	8
double		
zwindingmin	1x1	8
double		



Most of the data read in is not used in the present calculation. The TF contours are used and have been plotted above as a quick check. Next, type out the data just read in that is to be used here;

```
ITF
coil_name
[winding_r  winding_dr  winding_z  winding_dz  winding_mn]
coilname

ITF =
     4680000
coil_name =
    'PF1AU'
    'PF1BU'
    'PF1CU'
    'PF2U'
    ''
    'PF3U'
    ''
    'PF4'
    ''
    ''
    ''
    'PF5'
    ''
    ''
    ''
    'PF3L'
    ''
    'PF2L'
    ''
    'PF1CL'
    'PF1BL'
    'PF1AL'
    'OH'
```

```
    'PLASMA'
ans =
    0.3193    0.0593    1.5906    0.4635   64.0000
    0.4004    0.0336    1.8042    0.1812   32.0000
    0.5505    0.0373    1.8136    0.1664   20.0000
    0.8000    0.1627    1.9335    0.0680   14.0000
    0.8000    0.1627    1.8526    0.0680   14.0000
    1.4945    0.1864    1.6335    0.0680   15.0000
    1.4945    0.1864    1.5526    0.0680   15.0000
    1.7946    0.0915    0.8072    0.0680    8.0000
    1.8065    0.1153    0.8881    0.0680    9.0000
    1.7946    0.0915   -0.8072    0.0680    8.0000
    1.8065    0.1153   -0.8881    0.0680    9.0000
    2.0128    0.1353    0.6521    0.0686   12.0000
    2.0128    0.1353    0.5780    0.0686   12.0000
    2.0128    0.1353   -0.6521    0.0686   12.0000
    2.0128    0.1353   -0.5780    0.0686   12.0000
    1.4945    0.1864   -1.5526    0.0680   15.0000
    1.4945    0.1864   -1.6335    0.0680   15.0000
    0.8000    0.1627   -1.9335    0.0680   14.0000
    0.8000    0.1627   -1.8526    0.0680   14.0000
    0.5505    0.0373   -1.8136    0.1664   20.0000
    0.4004    0.0336   -1.8042    0.1812   32.0000
    0.3193    0.0593   -1.5906    0.4635   64.0000
    0.2421    0.0693         0    4.2416  884.0000
    1.0700    1.1000         0    2.0000    1.0000
coilname =
    'PF1AU'
    'PF1BU'
    'PF1CU'
    'PF2U'
    'PF3U'
    'PF4'
    'PF5'
    'PF3L'
    'PF2L'
    'PF1CL'
    'PF1BL'
    'PF1AL'
    'OH'
    'PLASMA'
```

Although the OH coil was supposed have 884 turns, it turns out that when it was wound it only 880 turns would fit. Therefore, the 884 will here be changed, turned into 880.

`winding_mn(23)=880`

```
winding_mn =
    64
    32
    20
    14
    14
    15
    15
     8
     9
```

```
     8
     9
    12
    12
    12
    12
    15
    15
    14
    14
    20
    32
    64
   880
     1
```

Next, we calculate the flux at each TF contour point produced by one ampere-turn in each of 24 "windings". To do so, two m-files will be invoked. In the interest of documenting the calculations, these two m-files first are copied as text, below.

```
function [r,z]=filamentize(rmin,rmax,zmin,zmax,m,n)
% Calculates filament matrix to represent rectangular xsection pf coils.
% Rectangle is divided into subrectangles; filaments are located at
% subrectangle centers.
%
r=zeros(m,n);z=r;
for j=1:n;
    r(:,j)=rmin+(rmax-rmin)/n*(j-0.5);
end
for i=1:m;
    z(i,:)=zmin+(zmax-zmin)/m*(i-0.5);
end
```

```
function [br,bz,flux]=poloidal_fieldy(rho,zeta,r,z);
% Axisymmetric poloidal magnetic field and flux are calculated
% at locations specified in matrices rho and zeta, normalized to a
% total source current of one ampere uniformly distributed between
% circular loop filament locations specified by matrices r and z.
% Matrix sizes must match between r and z which may be 1D or 2D arrays.
% Sizes of rho and zeta must also match each other but they may be ND
% arrays. SI units are used.
%
if size(r)~=size(z);
    error('r and z filament description matrices must be of same size')
end
if size(rho)~=size(zeta);
    error('rho and zeta field calc location description arrays must be of same size')
end
br=zeros(size(rho));bz=br;flux=br;aphi=flux;
%The following uses linear indexing for rho,zeta,br,bz,aphi,flux
%Note that aphi is the vector potential and flux is integrated over two pi radians.
for i=1:numel(br);
        rhot=rho(i);zetat=zeta(i);
% First confirm this evaluation location misses all filaments.
        disc=(r-rhot).^2+(z-zetat).^2;
        if (isempty( find( disc==0 ) ) );
        m=4*rhot*r./((rhot+r).^2+(zetat-z).^2);
        [K,E]=ellipke(m);
        bz(i)= mean(mean((2e-7).*(K+((r.^2-rhot^2-(zetat-z).^2)./disc).*E)...
        ./sqrt((rhot+r).^2.+(zetat-z).^2)  )');
        if rhot==0;
            br(i)=0.;
            aphi(i)=0.;
        else
```

```
            br(i)=mean(mean(2e-7*(zetat-z)./sqrt((rhot+r).^2+(zetat-z).^2).*(-K+...
            (r.^2+rhot^2+(zetat-z).^2)./disc.*E)/rhot)');
            aphi(i)=mean(mean(4e-7*sqrt(r/rhot).*((1.-m/2).*K-E)./sqrt(m))');
        end
        flux(i)=2*pi*rhot*aphi(i);
        else
            bz(i)=NaN;
            br(i)=NaN;
            aphi(i)=NaN;
            flux(i)=NaN;
        end
end
```

The magnetic calculations are done by replacing each coil winding pack by a set of circular filaments, where the spacing between filaments is approximately 2 cm.

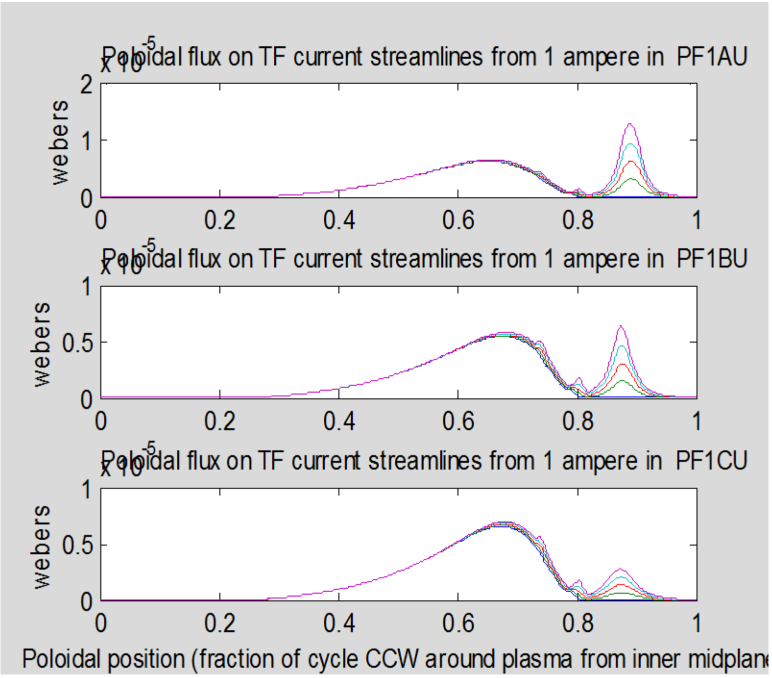
```
tilesize=0.02;
tiling=ceil([winding_dz winding_dr]/tilesize);
flux_from_windings=zeros(2000,5,24);
 for i=1:24;
[rf,zf]=filamentize(winding_r(i)-winding_dr(i)/2,
winding_r(i)+winding_dr(i)/2, winding_z(i)-winding_dz(i)/2,
winding_z(i)+winding_dz(i)/2, tiling(i,1),tiling(i,2) );
[brw,bzw,flux_from_windings(:,:,i)]=poloidal_fieldy(rC,zC,rf,zf);
end
```

Next, we calculate the fluxes for each circuit on the TF current streamline contour points, by summing the products of turns and flux per amp-turn over the windings series-connected within each circuit.

```
flux=zeros(2000, 5,14);
flux(:,:,1)=flux_from_windings(:,:,1)*winding_mn(1);
flux(:,:,2)=flux_from_windings(:,:,2)*winding_mn(2);
flux(:,:,3)=flux_from_windings(:,:,3)*winding_mn(3);
flux(:,:,4)=flux_from_windings(:,:,4)*winding_mn(4)+flux_from_windings(:
,:,5)*winding_mn(5);
flux(:,:,5)=flux_from_windings(:,:,6)*winding_mn(6)+flux_from_windings(:
,:,7)*winding_mn(7);
flux(:,:,6)=flux_from_windings(:,:,8)*winding_mn(8)+flux_from_windings(:
,:,9)*winding_mn(9)+flux_from_windings(:,:,10)*winding_mn(10)+flux_from_
windings(:,:,11)*winding_mn(11);
flux(:,:,7)=flux_from_windings(:,:,12)*winding_mn(12)+flux_from_windings
(:,:,13)*winding_mn(13)+flux_from_windings(:,:,14)*winding_mn(14)+flux_f
rom_windings(:,:,15)*winding_mn(15);
flux(:,:,8)=flux_from_windings(:,:,16)*winding_mn(16)+flux_from_windings
(:,:,17)*winding_mn(17);
flux(:,:,9)=flux_from_windings(:,:,18)*winding_mn(18)+flux_from_windings
(:,:,19)*winding_mn(19);
flux(:,:,10)=flux_from_windings(:,:,20)*winding_mn(20);
flux(:,:,11)=flux_from_windings(:,:,21)*winding_mn(21);
flux(:,:,12)=flux_from_windings(:,:,22)*winding_mn(22);
flux(:,:,13)=flux_from_windings(:,:,23)*winding_mn(23);
flux(:,:,14)=flux_from_windings(:,:,24)*winding_mn(24);
```

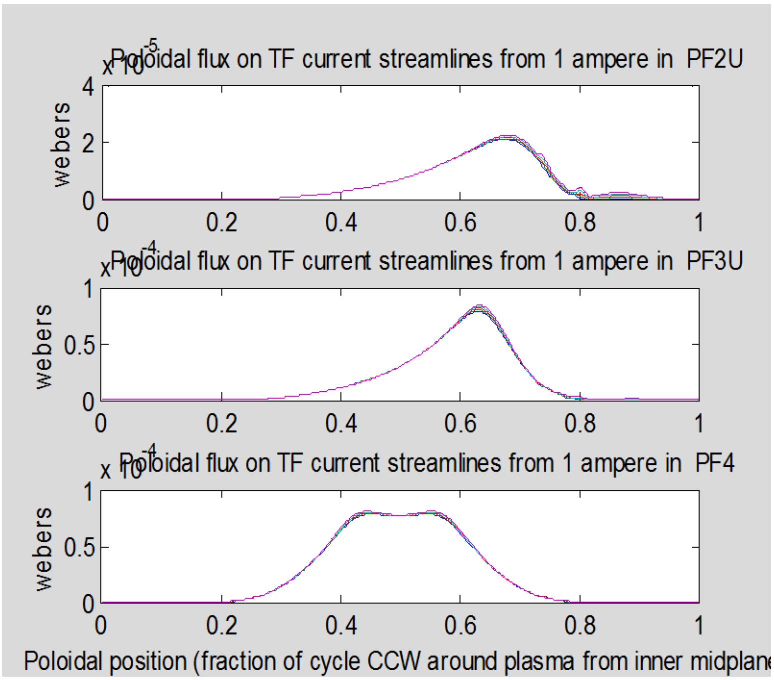
Next, make plots of the flux values varying about the poloidal circuits.

```
C=linspace(0,1,2000)';
clf;
i=1;
subplot(3,1,i);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ',char(coilname(i))] );
ylabel('webers');
i=2;
subplot(3,1,i);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
i=3;
subplot(3,1,i);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
xlabel('Poloidal position (fraction of cycle CCW around plasma from
inner midplane)')
```

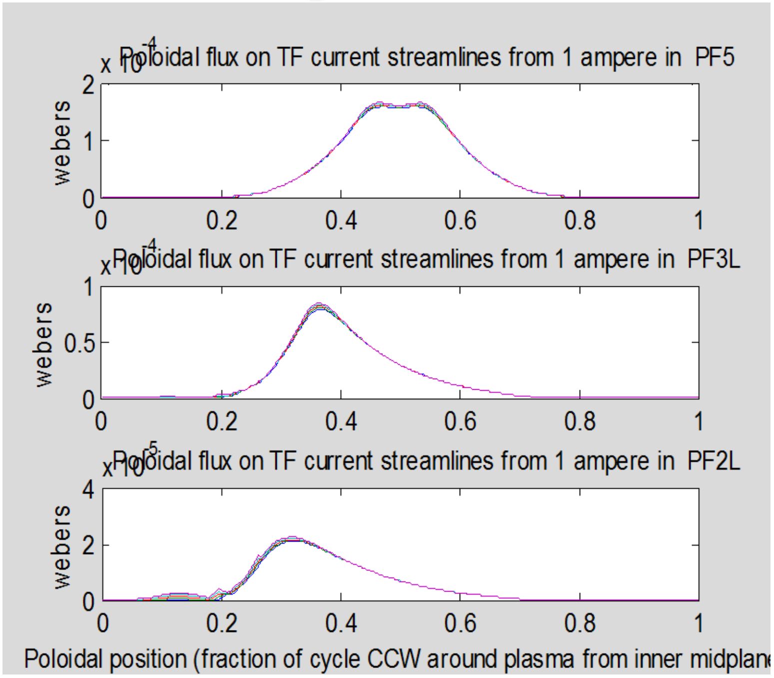



```
clf;
i=4;
subplot(3,1,i-3);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ',char(coilname(i))] );
ylabel('webers');
i=5;
```

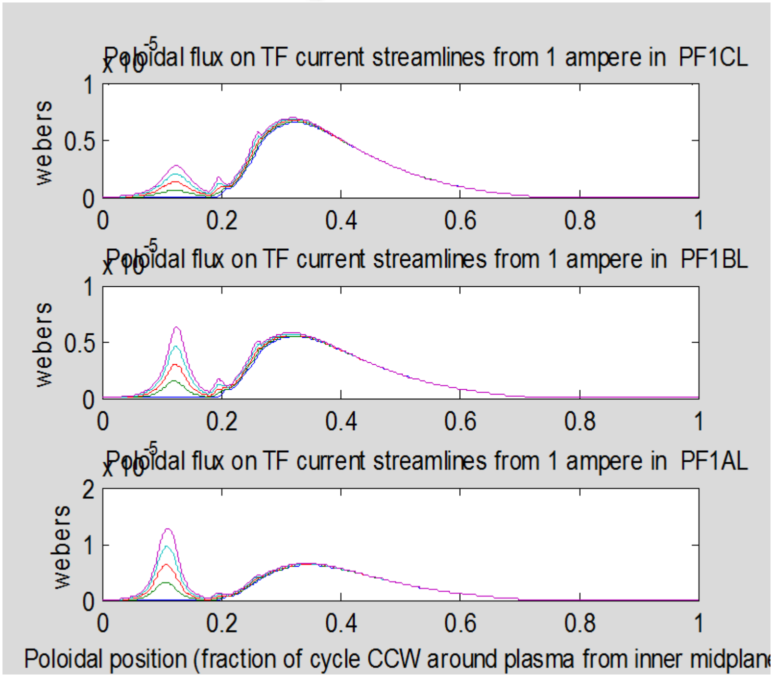
```
subplot(3,1,i-3);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
i=6;
subplot(3,1,i-3);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
xlabel('Poloidal position (fraction of cycle CCW around plasma from
inner midplane)')
```



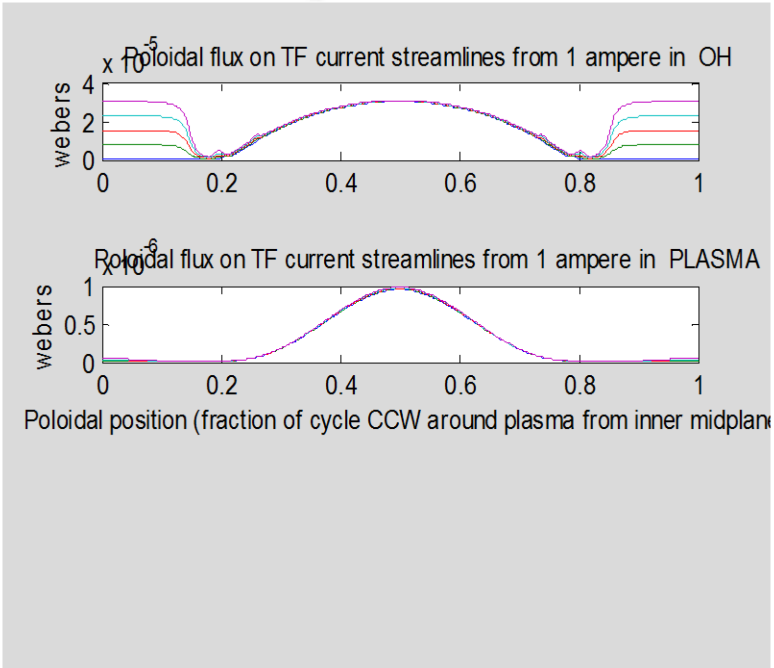

```
i=7;
subplot(3,1,i-6);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ',char(coilname(i))] );
ylabel('webers');
i=8;
subplot(3,1,i-6);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
i=9;
subplot(3,1,i-6);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
xlabel('Poloidal position (fraction of cycle CCW around plasma from
inner midplane)')
```




```
i=10;
subplot(3,1,i-9);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ',char(coilname(i))] );
ylabel('webers');
i=11;
subplot(3,1,i-9);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
i=12;
subplot(3,1,i-9);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
xlabel('Poloidal position (fraction of cycle CCW around plasma from
inner midplane)')
```




```
i=13;clf;
subplot(3,1,i-12);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ',char(coilname(i))] );
ylabel('webers');
i=14;
subplot(3,1,i-12);plot(C,flux(:,:,i));title(['     Poloidal flux on TF
current streamlines from 1 ampere in  ', char(coilname(i))] );
ylabel('webers')
xlabel('Poloidal position (fraction of cycle CCW around plasma from
inner midplane)')
```



Next, form the means over the TF thickness:

```
mean_flux=zeros(2000,14);
for i=1:14;
mean_flux(:,i)=flux(:,:,i)*[1 2 2 2 1]'/8;
end
```

Cumulative torsion per ampere squared (i.e., per total TF amp per PF ckt amp) is then the local flux referenced to the TF coil starting point divided by 2π.

```
for i=1:14;
cum_torsion_per_amp2(:,i) =(mean_flux(:,i)-mean_flux(1,i))/2/pi;
local_torsion_density_per_amp2(1:(end-1),i)=diff(
cum_torsion_per_amp2(:,i) )*1999.;
local_torsion_density_per_amp2(end,i)=local_torsion_density_per_amp2(1,i
);
end
```

At this point in the calculations, we have the cumulative torsion per amp squared as a function of the node number , for each of 14 PF currents. This is the point at which the coefficients that I am looking for should be extracted.

```
format long g
keynodes=[566 1000 1001 1435]';
cum_torque=ITF*1000*cum_torsion_per_amp2(keynodes,:);
cum_torque= [1 0 0 0; 0 0.5 0.5 0; 0 0 0 1]*cum_torque;
coilname
OuterLegTorqueCoefs=cum_torque(3,:)'-cum_torque(1,:)'
UpperHalfTorqueCoefs=-cum_torque(2,:)'
UpperOuterTorqueCoefs=cum_torque(3,:)'-cum_torque(2,:)'
LowerOuterTorqueCoefs=cum_torque(2,:)'-cum_torque(1,:)'
```

```
coilname =
    'PF1AU'
    'PF1BU'
    'PF1CU'
    'PF2U'
    'PF3U'
    'PF4'
    'PF5'
    'PF3L'
    'PF2L'
    'PF1CL'
    'PF1BL'
    'PF1AL'
    'OH'
    'PLASMA'
OuterLegTorqueCoefs =
          3518.87969398684
          3691.47402276291
          4293.43260812891
          13190.4182202296
          16497.6103914842
         0.112063331722311
         0.155651822782602
         -16497.2884931156
          -13190.266095538
         -4293.39257586766
         -3691.44357481106
         -3518.84262872701
         0.101191143633514
       0.00143743023065213
UpperHalfTorqueCoefs =
         -2260.22563374338
         -1580.33378737855
         -1851.32006614982
         -5197.71673166169
          -22045.642269688
         -57019.4694938823
         -118638.041645382
         -22045.9545355283
         -5197.78563720586
         -1851.34393062627
         -1580.35395505113
         -2260.25305753874
         -11491.3317357217
         -705.500762395247
UpperOuterTorqueCoefs =
          1351.96326743527
          2181.23794943332
          2525.15220014378
          8232.35295369789
         -4528.02679162464
         -50563.9779114056
         -109651.322322954
         -21025.9303982053
         -4958.12945555722
```

```
         -1768.30255898383
         -1510.25476701242
         -2166.94166368394
         -12462.5950934212
         -634.845510358332
LowerOuterTorqueCoefs =
          2166.91642655157
          1510.23607332959
          1768.28040798513
          4958.06526653175
          21025.6371831089
          50564.0899747373
          109651.477974777
          4528.64190508965
         -8232.13663998076
         -2525.09001688383
         -2181.18880779863
         -1351.90096504307
          12462.6962845648
          634.846947788563
```

Based on these calculated results, the torque algorithms to be used for coil system protection are written as follows:

$$\left[\frac{\text{Net TF System OuterLeg Torque}}{1\,\text{N-m}}\right]=\left[\frac{I_{TF}}{130\,\text{kA}}\right]\left(\begin{array}{l}3518.8\left[\dfrac{I_{PF1AU}-I_{PF1AL}}{1\,\text{kA}}\right]\\+3691.4\left[\dfrac{I_{PF1BU}-I_{PF1BL}}{1\,\text{kA}}\right]\\+4293.4\left[\dfrac{I_{PF1CU}-I_{PF1CL}}{1\,\text{kA}}\right]\\+13190.4\left[\dfrac{I_{PF2U}-I_{PF2L}}{1\,\text{kA}}\right]\\+16497.4\left[\dfrac{I_{PF3U}-I_{PF3L}}{1\,\text{kA}}\right]\end{array}\right)$$

$$\left[\frac{\text{Net Upper Half TF System Torque}}{1\,\text{N-m}}\right]$$

$$=-\left[\frac{I_{TF}}{130\,\text{kA}}\right]\left(\begin{array}{l}11491.3\left[\dfrac{I_{OH}}{1\,\text{kA}}\right]+2260.2\left[\dfrac{I_{PF1AU}+I_{PF1AL}}{1\,\text{kA}}\right]+1580.3\left[\dfrac{I_{PF1BU}+I_{PF1BL}}{1\,\text{kA}}\right]\\+1851.3\left[\dfrac{I_{PF1CU}+I_{PF1CL}}{1\,\text{kA}}\right]+5197.7\left[\dfrac{I_{PF2U}+I_{PF2L}}{1\,\text{kA}}\right]\\+22045.8\left[\dfrac{I_{PF3U}+I_{PF3L}}{1\,\text{kA}}\right]+57019.5\left[\dfrac{I_{PF4}}{1\,\text{kA}}\right]\\+118638.0\left[\dfrac{I_{PF5}}{1\,\text{kA}}\right]+705501\left[\dfrac{I_{PLASMA}}{1\,\text{MA}}\right]\end{array}\right)$$

Note that the OH coeffidient above is much different from the coefficient in my 11 February 2011 memo. Reason is likely that the older coefficient reflected a different number of turns, over 1000 OH turns, whereas the above value is for 880 turns.

$$\left[\frac{\text{Net Upper Outer TF System Torque}}{1\,\text{N-m}}\right]$$

$$=\left[\frac{I_{TF}}{130\,\text{kA}}\right]\left(\begin{array}{l} +1351.9\left[\frac{I_{PF1AU}}{1\,\text{kA}}\right]+2181.2\left[\frac{I_{PF1BU}}{1\,\text{kA}}\right] \\ +2525.1\left[\frac{I_{PF1CU}}{1\,\text{kA}}\right]+8232.2\left[\frac{I_{PF2U}}{1\,\text{kA}}\right] \\ -4528.3\left[\frac{I_{PF3U}}{1\,\text{kA}}\right]-50564.\left[\frac{I_{PF4}}{1\,\text{kA}}\right] \\ -109651.4\left[\frac{I_{PF5}}{1\,\text{kA}}\right]-21026.7\left[\frac{I_{PF3L}}{1\,\text{kA}}\right] \\ -4958.2\left[\frac{I_{PF2L}}{1\,\text{kA}}\right]-1768.3\left[\frac{I_{PF1CL}}{1\,\text{kA}}\right] \\ -1510.2\left[\frac{I_{PF1BL}}{1\,\text{kA}}\right]-2166.9\left[\frac{I_{PF1AL}}{1\,\text{kA}}\right] \\ -12462.6\left[\frac{I_{OH}}{1\,\text{kA}}\right]-634845.5\left[\frac{I_{PLASMA}}{1\,\text{MA}}\right] \end{array}\right)$$

$$\left[\frac{\text{Net Lower Outer TF System Torque}}{1\,\text{N-m}}\right]$$

$$=\left[\frac{I_{TF}}{130\,\text{kA}}\right]\left(\begin{array}{l} +2166.9\left[\frac{I_{PF1AU}}{1\,\text{kA}}\right]+1510.2\left[\frac{I_{PF1BU}}{1\,\text{kA}}\right] \\ +1768.3\left[\frac{I_{PF1CU}}{1\,\text{kA}}\right]+4958.2\left[\frac{I_{PF2U}}{1\,\text{kA}}\right] \\ 21025.7\left[\frac{I_{PF3U}}{1\,\text{kA}}\right]+50564.\left[\frac{I_{PF4}}{1\,\text{kA}}\right] \\ +109651.4\left[\frac{I_{PF5}}{1\,\text{kA}}\right]+4528.3\left[\frac{I_{PF3L}}{1\,\text{kA}}\right] \\ -8232.2\left[\frac{I_{PF2L}}{1\,\text{kA}}\right]-2525.1\left[\frac{I_{PF1CL}}{1\,\text{kA}}\right] \\ -2181.2\left[\frac{I_{PF1BL}}{1\,\text{kA}}\right]-1351.9\left[\frac{I_{PF1AL}}{1\,\text{kA}}\right] \\ +12462.6\left[\frac{I_{OH}}{1\,\text{kA}}\right]+634845.5\left[\frac{I_{PLASMA}}{1\,\text{MA}}\right] \end{array}\right)$$

----

save data in C:\Program Files\MATLAB\R2014a

```
save NSTXU_Torque_Coefs_Data_13Feb2015
```

Putting these coefs into a table format, they are as follows:

New NSTX-U TF Torque Coefficients 13 February 2015

PF\TorquedPart	OuterLeg	-UpperHalf	UpperOuter	LowerOuter
'PF1AU'	3518.8	2260.2	1351.9	2166.9
'PF1BU'	3691.4	1580.3	2181.2	1510.2
'PF1CU'	4293.4	1851.3	2525.1	1768.3
'PF2U'	13190.4	5197.7	8232.2	4958.2
'PF3U'	16497.4	22045.8	-4528.3	21025.7
'PF4'	0	57019.5	-50564.	50564.
'PF5'	0	118638.0	-109651.4	109651.4
'PF3L'	-16497.4	22045.8	-21026.7	4528.3
'PF2L'	-13190.4	5197.7	-4958.2	-8232.2
'PF1CL'	-4293.4	1851.3	-1768.3	-2525.1
'PF1BL'	-3691.4	1580.3	-1510.2	-2181.2
'PF1AL'	-3518.8	2260.2	-2166.9	-1351.9
'OH'	0	11491.3	-12462.6	12462.6
'PLASMA'	0	705501	-634846.3	634846.3

It is useful to compare the new coefficient values with previous values for the first two columns above:

PF\TorquedPart	OuterLeg(new)	OuterLeg(old)	RATIO New/Old
'PF1AU'	3518.8	**3519.9**	**0.9997**
'PF1BU'	3691.4	**3692.0**	**0.9998**
'PF1CU'	4293.4	**4293.8**	**0.9999**
'PF2U'	13190.4	**13191.**	**0.99995**
'PF3U'	16497.4	**16497**	**1.00002**
'PF4'	0	**0**	**?**
'PF5'	0	**0**	**?**
'PF3L'	-16497.4	**-16497**	**1.00002**
'PF2L'	-13190.4	**-13191**	**0.99995**
'PF1CL'	-4293.4	**-4293.8**	**0.9999**
'PF1BL'	-3691.4	**-3692.0**	**0.9998**
'PF1AL'	-3518.8	**-3519.9**	**0.9997**
'OH'	0	**0**	**?**
'PLASMA'	0	**0**	**?**

For the upper half, the comparison table is as follows:

PF\TorquedPart	-UpperHalf (new)	Upper Half (old)	RATIO New/Old
'PF1AU'	2260.2	**2260.9**	**0.9997**
'PF1BU'	1580.3	**1580.6**	**0.9998**
'PF1CU'	1851.3	**1851.5**	**0.9999**
'PF2U'	5197.7	**5197.5**	**1.00004**
'PF3U'	22045.8	**21915.7**	**1.006**
'PF4'	57019.5	**56813.9**	**1.0036**
'PF5'	118638.0	**118636.5**	**1.00001**
'PF3L'	22045.8	**21915.7**	**1.006**
'PF2L'	5197.7	**5197.5**	**1.00004**
'PF1CL'	1851.3	**1851.5**	**0.9999**
'PF1BL'	1580.3	**1580.6**	**0.9998**
'PF1AL'	2260.2	**2260.9**	**0.9997**
'OH'	11491.3	**13563.1**	**0.84725**
'PLASMA'	705501	**713308.9**	**0.9891**

The tables show that all ratios between new and old torque coefficients are very close to unity except for one. i.e., the OH coil. Investigation into the "old" value's calculation, which was performed prior to February 11, 2011, revealed that it had used the previous OH coil design's larger number of turns.

13-180911

To: Distribution Date: 18 September 2011

From: R. Woolley Subject TF Inner Leg Shear Stress in NSTX CSU

References:

1. **R. Woolley memo 13-260709, "Out-Of-Plane (OOP) PF/TF Torques On TF Conductors in NSTX CSU", 26 July 2009,**
2. **P. Titus memo, "Maximum TF Torsional Shear", 29 July 2009**
3. **P. Titus paper, "Provisions for Out-of-Plane Support of the TF Coils in Recent Tokamaks",28September1999**
4. **R. Woolley memo 13-110211, "Torques On TF Conductors & Resulting Torsion & Shear Stress in NSTX CSU, 04May2010 Design Point", 11 February 2011**
5. **P. Titus, NSTXU*-CALC-132-07-00 Rev0, "TF Inner Leg Torsional Shear, Including Input to the DCPS", 9Sept2011**

Summary

This memo replaces Ref.4. It results from my review of P. Titus' Ref.5 and comparisons of its predicted shear stress results with those of my own Ref.4 analyses. Findings are as follows:

(1) Ref.5 is confirmed as following correct calculation methods and should therefore be just as accurate as the Global Model on which it is based. (The Global Model review is by others.)

(2) My Ref.4 calculations contained an error which had the effect of underestimating structural supporting torques, thus resulting in distorted torsional shear distributions within the TF inner leg. The present memo corrects this error (see Eqs.27) and documents the resulting shear stress distributions for NSTX-U's 96 prescribed plasma equilibria and for its OH precharge condition.

(3) Maximum local shear stress (a) occurs when PF coil currents are combined with maximally negative OH current, and (b) are spatially localized in small regions near the ends of the inner leg before reaching the lead extensions. This qualitatively matches the behavior predicted in Ref.5.

(4) Peak local shear stress of **25.234 MPa** is predicted for equilibria #1 and #16, using nominal stiffness parameters estimated for the NSTX design as of December 2010. (Note that this peak value only applies at a point location; any finite region has a lower average.) The 97 shear stress distributions for the inner leg are documented in Appendix 2 (i.e., page 50).

(5) An effort was undertaken to investigate whether deliberate changes in stiffnesses could reduce peak torsional shear stress. It was found that ***reduction in stiffness*** of either umbrella lid reduces peak inner leg shear. For example, a 95% reduction of upper lid stiffness reduces peak local shear stress in the inner leg by 22.5% to **19.553 MPa**. The 97 shear stress distributions for the inner leg with this reduced upper lid stiffness are documented in Appendix 3 (i.e., page 70).

As in Ref.4, this memo advances and uses a simple algorithm based on magnetic flux differences for evaluating out-of-plane torques due to magnetic interactions of poloidal magnetic fields with TF conductor current. This is the subject of Appendix 1 (i.e., page 42). The torsional elastic response model is the subject of pages 2-40. Analysis results are discussed on page 41.

OOP Torque Analysis of NSTX CSU

As in Refs.1 & 4, the NSTX CSU TF conductor shape definition was provided by Peter Titus on 26 June 2009 in a file containing XYZ 3D coordinates of 4229 nodal points delimiting hexahedral elements in a 30 degree sector global TF model. These were subsequently culled and sorted by various automated MATLAB methods to obtain a poloidal half-plane outline consisting of 322 points, including an inner outline trace of 159 points and an outer outline trace of 163 points. These points were only modified from the provided file by setting their third coordinate values to zero without changing their other two coordinates. The resulting TF conductor outline appears in the following Fig.(1) MATLAB plot.

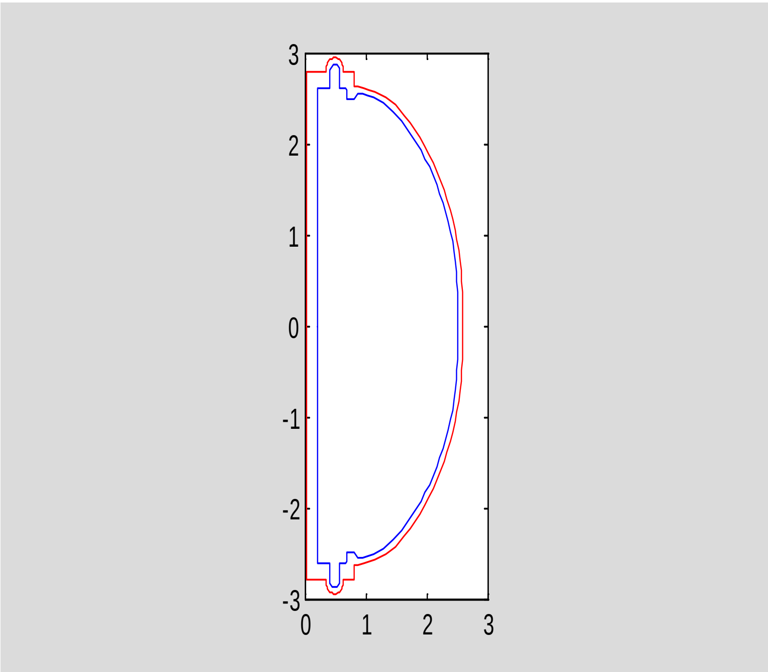


Figure 1: Poloidal Projection Outline of TF Conductor, 322 points

Parametric variables were then calculated and saved for each point in each of the two outline traces, starting from zero at their radially inner points on the (z=0) horizontal midplane and adding the distances to each successive point while proceeding in the counterclockwise direction. After returning to the starting points, the saved parametric values were then normalized by dividing each cumulative distance value by its contour's total perimeter length. The resulting parametric variables, which range from 0 to 1, roughly represent poloidal angle.

Finally, 2000 uniformly spaced values of this poloidal angle variable ranging from 0 to 1 were generated and 2000 corresponding r and z coordinate values were obtained for each of the TF coil's inner and outer outline curves by using MATLAB's standard interpolation m-file subroutines. The two resulting 2000-point outlines are plotted below in Fig. (2). Note that although they appear almost identical to the previous TF outline plot, each outline here consists of 2000 equally spaced points which are now linked to each other through their common normalized poloidal "angle" location parametric variable values. In each contour of this model, node point number 1 is also number 2000, located at Z=0 in the TF centerstack. Constant radius points on the centerstack's outer edge include 1 through 363 at the bottom and from 1638 at the top through 2000. TF leads extending to the flexes encompass nodes out to 371 and to 1630.

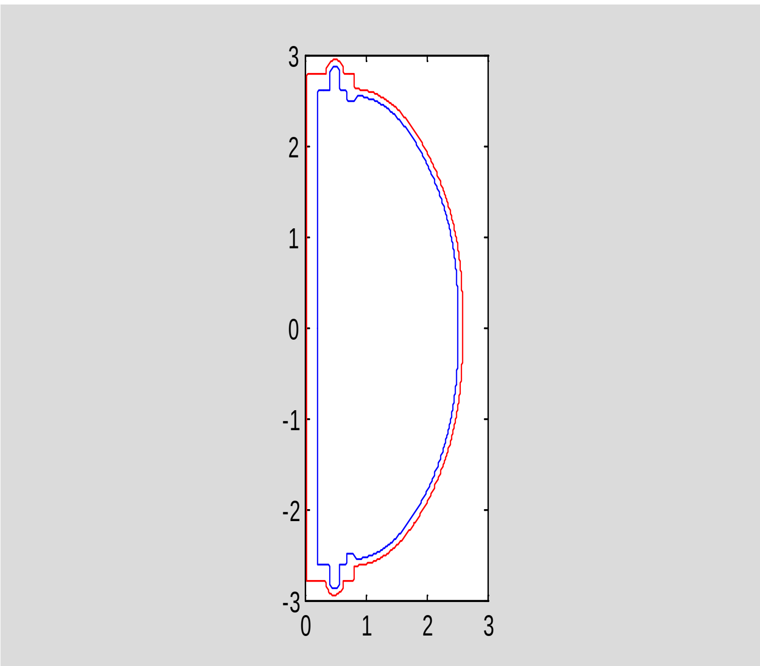


Figure 2: Poloidal Projection Outline of TF Conductor, 4000 points

Note that the TF current stream function which varies with the (r,z) location in the poloidal half-plane is defined as the total TF system current that passes through the 3D circle sharing those cylindrical coordinates. It varies from zero at (r,z) locations not linking the TF coils to the total TF current (i.e., number of turns times current per turn) at locations within the TF coil system bore. At intermediate locations within the TF conductor it varies between those values. Level set contours of the TF current stream function are also streamlines of the TF current flow.

As plotted, the (red) outer outline is the zero value contour of the TF current stream function and the (blue) inner outline is the contour of the TF current stream function at its full, 100% of TF system current, value. It was decided for calculations herein to try to improve accuracy by estimating how the TF current is distributed within the TF conductor. This was done by estimating the (r,z) coordinate locations at each of 2000 poloidal variable points for TF current stream function contours enclosing 25%, 50%, and 75%, respectively, of the total TF current. Ideally these contour estimates would be obtained by solving conductive media equations using ohms law, but this was not done. Instead, the estimated contours were obtained by interpolation between the outline coordinates for common poloidal angle variables. For the vertical (z) coordinates, linear interpolation was used directly. However, a different method was used to interpolate the r coordinate since conductor thickness in the toroidal direction varies in proportion to r for locations in and near the TF central bundle but takes on constant thickness for outer locations. Linear interpolation of a nonlinear function was used, where that function switched from a quadratic for inner locations to a straight line for outer locations. For this purpose the switchover radius between variable and constant toroidal thickness was estimated as r=0.3339 meters, based on inspection of plots showing the 4229 points of the global model.

The Fig.(3) plot follows showing the resulting estimated three internal contours and the two bounding contours of the TF current stream function.

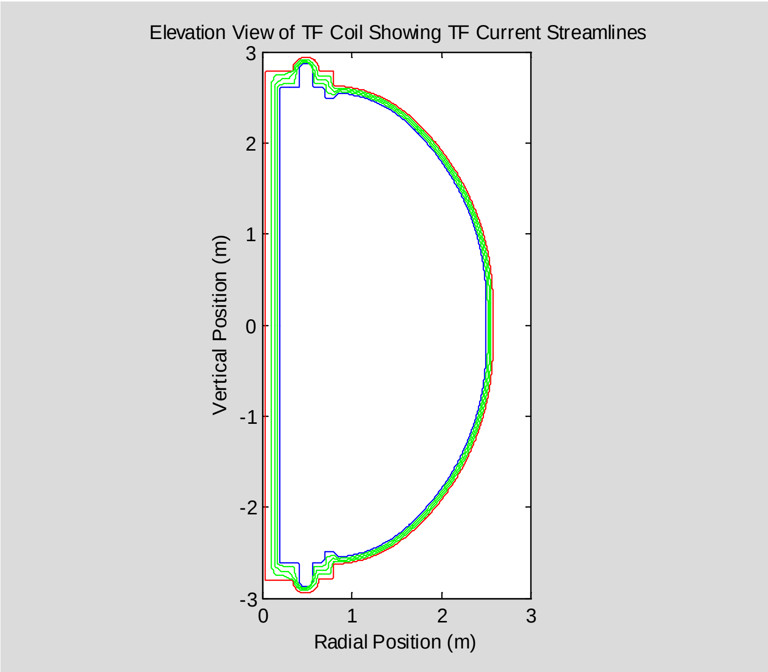


Figure 3: Poloidal Projection of TF Current Stream Function Contours, 10000 points

It may be useful to have plots showing r and z for the bounding outlines and internal contours as functions of the poloidal angle parametric variable. These follow:

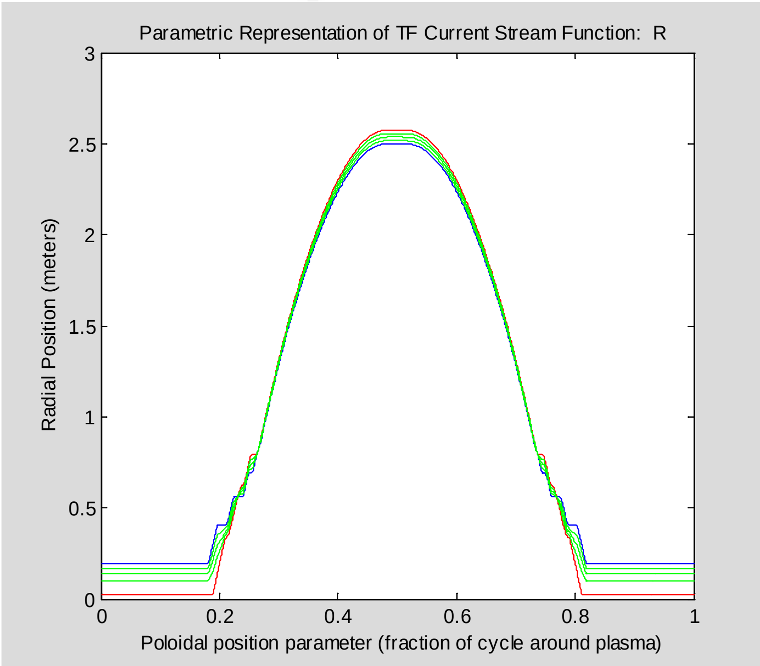


Figure 4: Radial Coordinates of TF Current Stream Function Contours vs Poloidal Variable

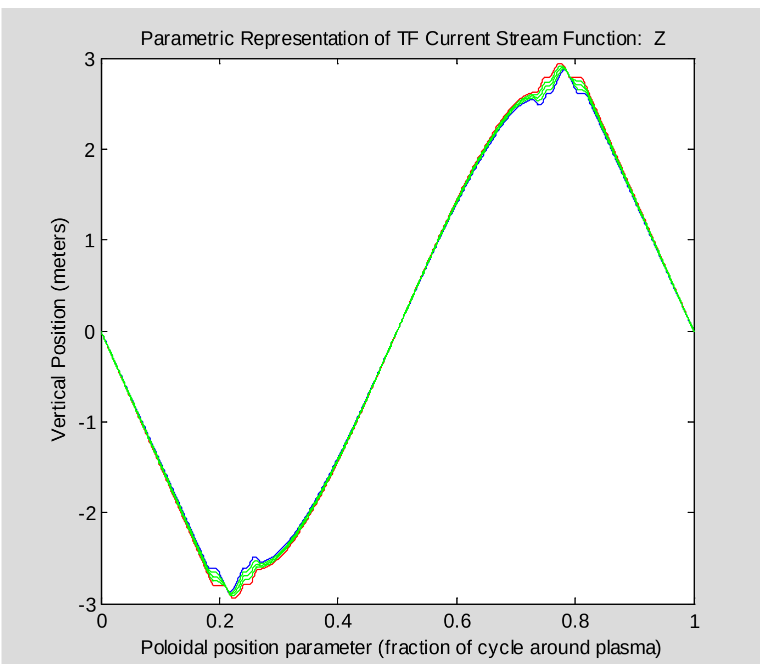


Figure 5: Vertical Coordinates of TF Current Stream Function Contours vs Poloidal Variable

The magnetics designs assumed herein for the OH coil and for the six coils, PF1AU, PF1BU, PF1CU, PF1CL, PF1BL, and PF1AL were adopted 04May2010. Later changes have been minor, and have not been explicitly addressed. Details of the poloidal field system for the 04May2010 design point have been reproduced here as Table 1 and are depicted in Fig.6. These include 23 coil windings that are configured into 13 series-connected circuits. They also include a crude. approximate magnetics model representing the plasma as a rectangular cross section uniform current density single-turn coil winding.

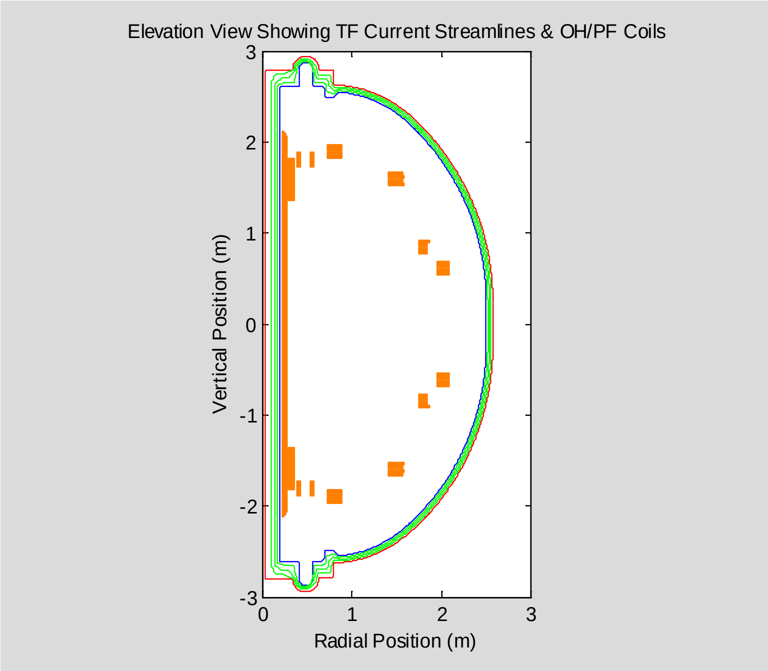


Figure 6: Poloidal Projection of PF&OH coils, with TF Current Stream Function Contours

Table 1: 04May2010 Design Point Poloidal Field System for PF Coils, OH Coil, Plasma (As Used For Magnetics Calculations)

Coil Power	Series-connected Winding Geometry					Rectangular Matrix of Turns		
Circuit Name	Rectangular Winding Name	R (center) cm	ΔR cm	Z (center) cm	ΔZ cm	# in z direction	# in r direction	Total # turns
PF1AU	PF1AU	31.93	5.9268	159.06	46.3533	16	4	64
PF1BU	PF1BU	40.038	3.36	180.42	18.1167	16	2	32
PF1CU	PF1CU	55.052	3.7258	181.36	16.6379	10	2	20
	PF2AU	79.9998	16.2712	193.3473	6.797	2	7	14
PF2U	PF2BU	79.9998	16.2712	185.26	6.797	2	7	14
	PF3AU	149.446	18.6436	163.3474	6.797	2	7.5	15
PF3U	PF3BU	149.446	18.6436	155.26	6.797	2	7.5	15
	PF4BU	179.4612	9.1542	80.7212	6.797	4	2	8
	PF4CU	180.6473	11.5265	88.8086	6.797	2	4.5	9
	PF4BL	179.4612	9.1542	-80.7212	6.797	4	2	8
PF4	PF4CL	180.6473	11.5265	-88.8086	6.797	2	4.5	9
	PF5AU	201.2798	13.5331	65.2069	6.858	2	6	12
	PF5BU	201.2798	13.5331	57.8002	6.858	2	6	12
	PF5AL	201.2798	13.5331	-65.2069	6.858	2	6	12
PF5	PF5BL	201.2798	13.5331	-57.8002	6.858	2	6	12
	PF3BL	149.446	18.6436	-155.26	6.797	2	7.5	15
PF3L	PF3AL	149.446	18.6436	-163.347	6.797	2	7.5	15
	PF2LA	79.9998	16.2712	-193.347	6.797	2	7	14
PF2L	PF2LB	79.9998	16.2712	-185.26	6.797	2	7	14
PF1CL	PF1CL	55.052	3.7258	-181.36	16.6379	10	2	20
PF1BL	PF1BL	40.038	3.36	-180.42	18.1167	16	2	32
PF1AL	PF1AL	31.93	5.9268	-159.06	46.3533	16	4	64
OH	OH	24.2083	6.934	0	424.16	221	4	884
PLASMA	PL	107.000	110.000	0.000	200.000	1	1	1

Next, poloidal flux was calculated for each of the 2000 locations on each of the five contours for each of the 13 coil circuits and for a crude electromagnetic model of a "plasma" having uniform current density and a rectangular cross section extending over $0.52<r<1.63$ and $-1<z<1$, all in meters. Plots of the computed poloidal magnetic fluxes appear below in Figs.(7)-(11).

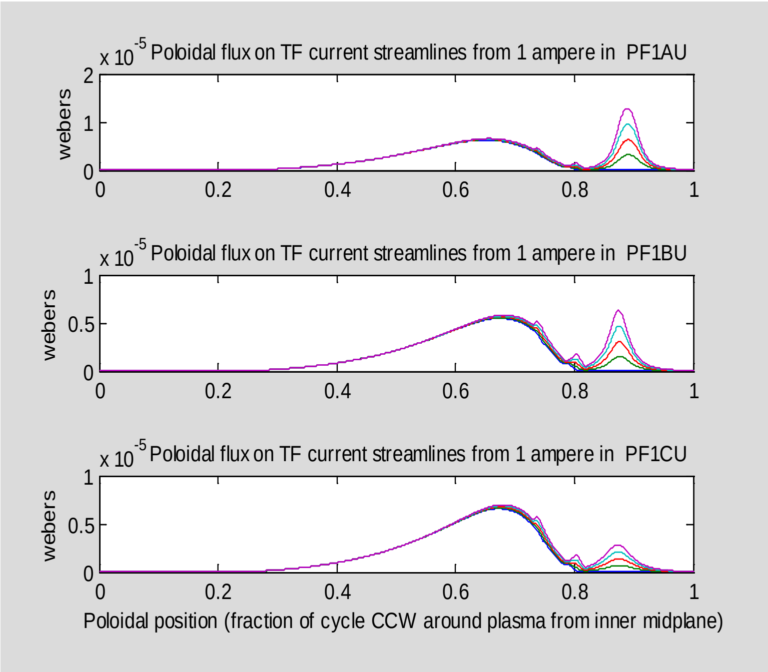


Figure 7: Poloidal Flux Per Ampere Excitation in Circuits 1-3

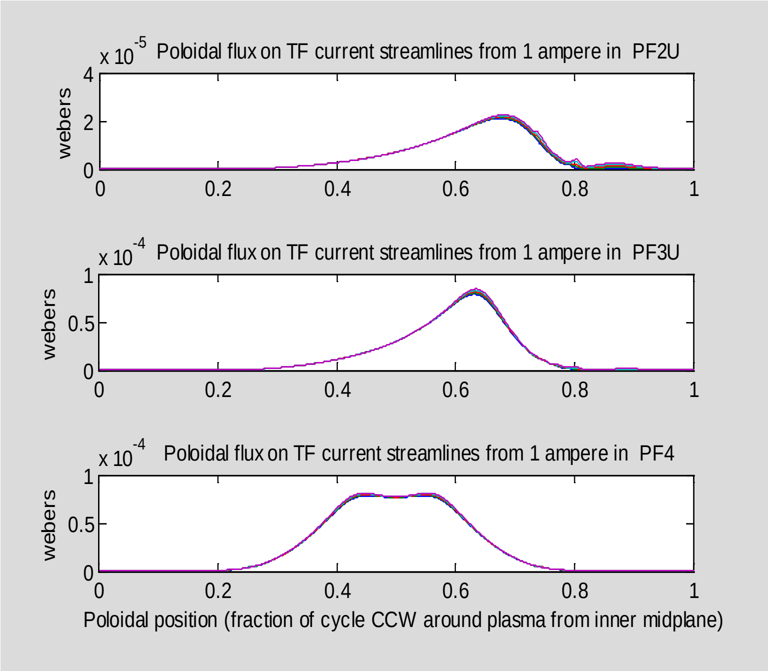


Figure 8: Poloidal Flux Per Ampere Excitation in Circuits 4-6

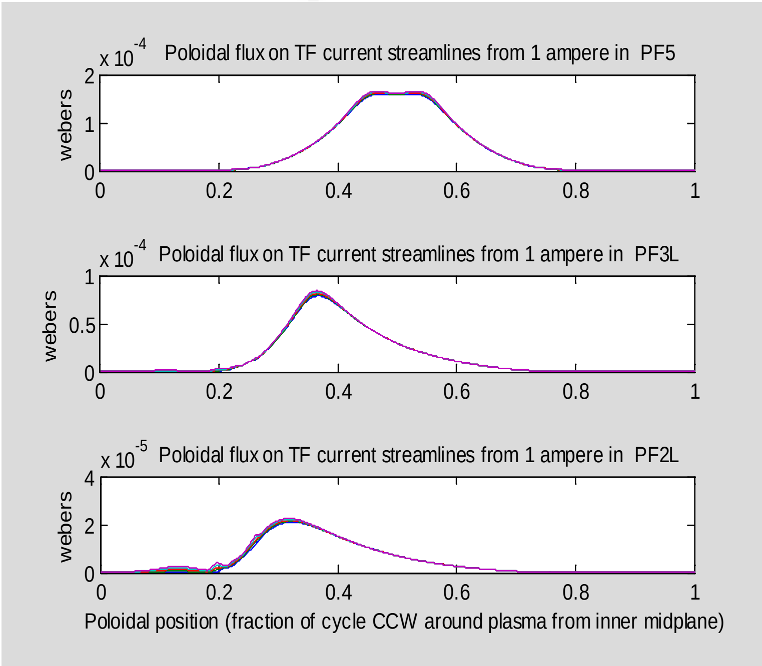


Figure 9: Poloidal Flux Per Ampere Excitation in Circuits 7-9

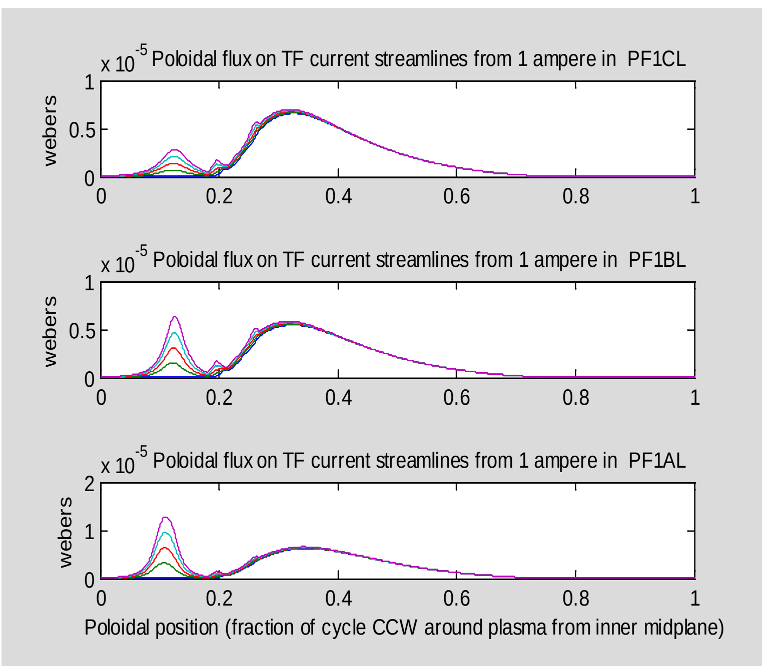


Figure 10: Poloidal Flux Per Ampere Excitation in Circuits 10-12

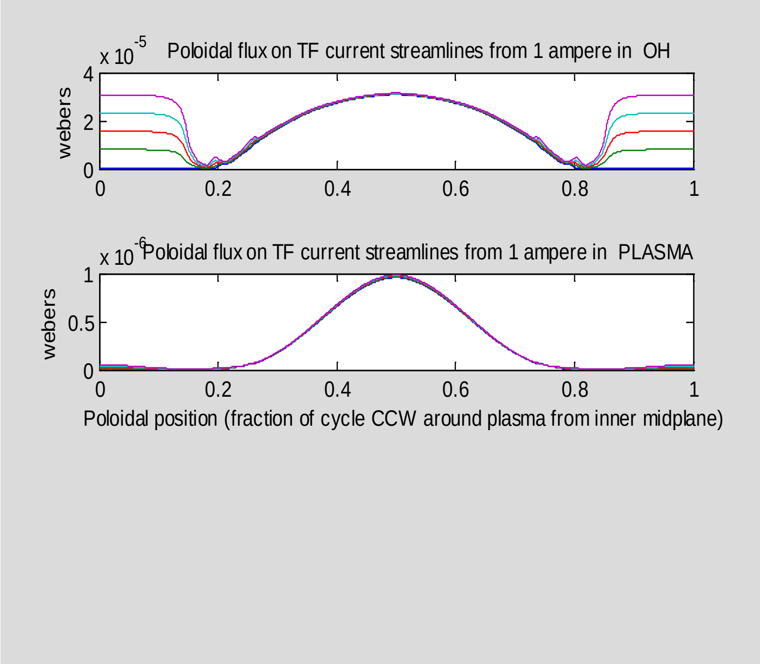


Figure 11: Poloidal Flux Per Ampere Excitation in Circuits 13-14

For each coil circuit, the mean flux was next calculated for each poloidal location as a weighted average of the flux values on the five current stream function contours, using weighting factors [0.125 0.25 0.25 0.25 0.125]. Local torque densities per ampere of circuit current are then evaluated as the product of total TF current times the finite difference derivative with respect to the poloidal angle variable of the mean flux divided by 2π. The resulting profiles were multiplied respectively by the Table 2 maximum and minimum circuit currents to establish the ranges of torque densities versus poloidal locations. Results appear in Figs.12-16.

Table 2: Coil And Plasma Current Ranges Assumed for Torque Load Range Calculations

COIL CIRCUIT NAME	MINIMUM CURRENT (Amperes)	MAXIMUM CURRENT (Amperes)
'PF1AU'	-7000	18000
'PF1BU'	0	13000
'PF1CU'	0	16000
'PF2U'	-11000	15000
'PF3U'	-16000	12000
'PF4'	0	16000
'PF5'	0	34000
'PF3L'	-16000	12000
'PF2L'	-11000	15000
'PF1CL'	0	16000
'PF1BL'	0	13000
'PF1AL'	-7000	18000
'OH'	-24000	24000
'PLASMA'	0	2000000

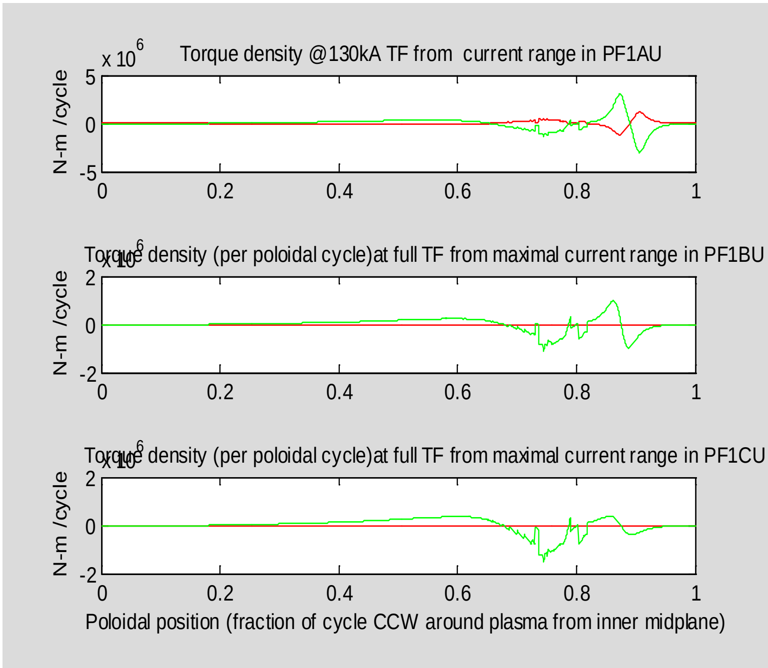


Figure 12: Torque Density For Maximal Range of Currents in Circuits 1-3

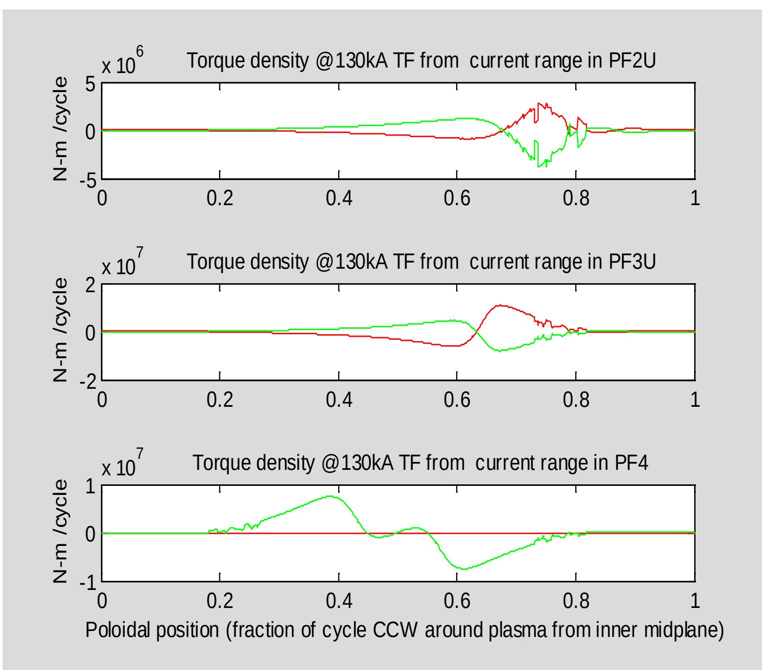


Figure 13: Torque Density For Maximal Range of Currents in Circuits 4-6

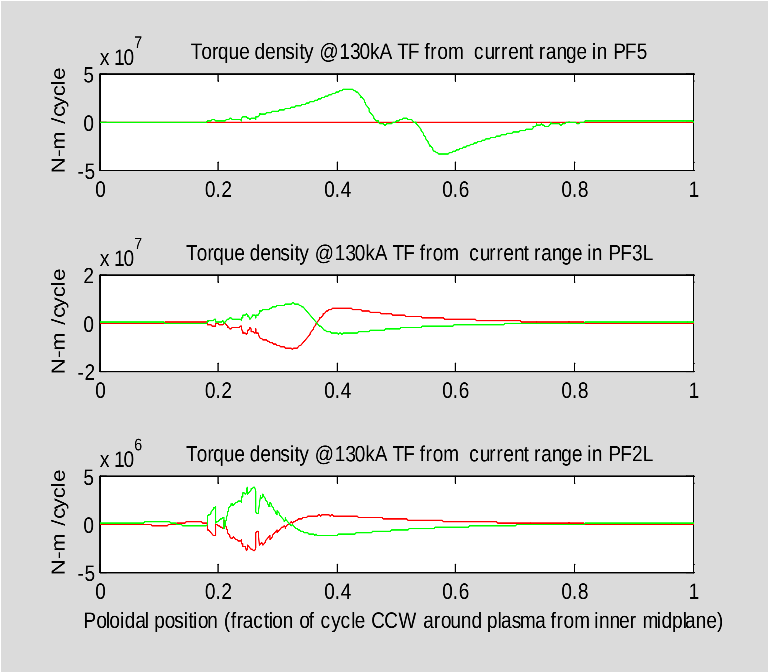


Figure 14: Torque Density For Maximal Range of Currents in Circuits 7-9

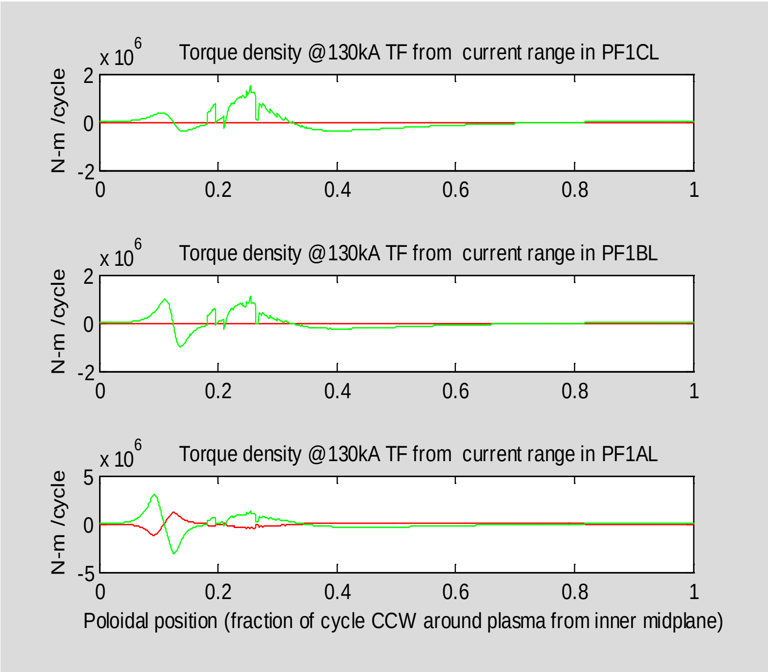


Figure 15: Torque Density For Maximal Range of Currents in Circuits 10-12

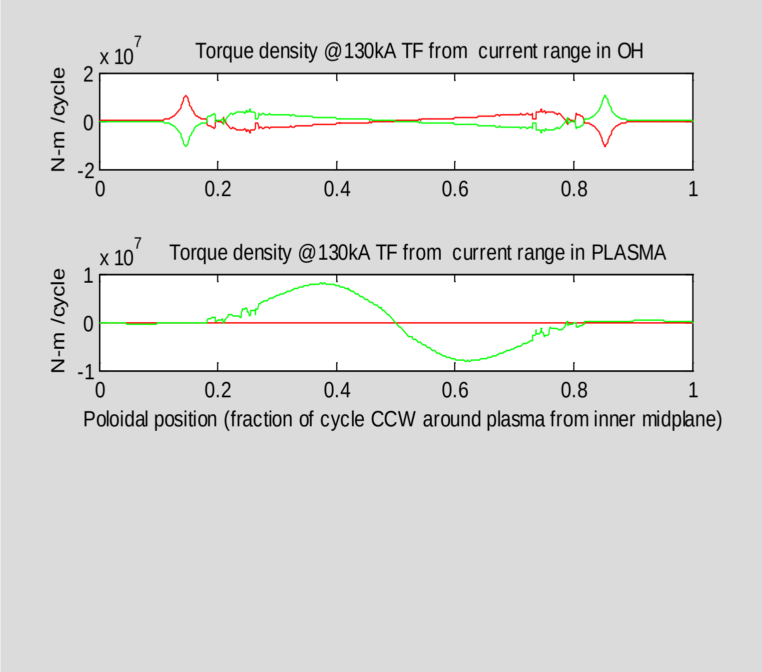


Figure 16: Torque Density For Maximal Range of Currents in Circuits 13-14

TORSIONAL RESPONSE ALGORITHMS: Overview

Although magnetic Lorenz torque loading on the TF coil system is completely determined by its electrical currents and magnetic fields, the torsion response of the system also reflects the mechanical propagation of torques and rotational deflections between different conductor assemblies and structural supports. The ***internal torque*** state at any imaginary section cut through the TF conductor system is defined as the external torque that would need to be applied to one side of that cut to maintain its mechanical equilibrium. This torque is oppositely directed for the two sides of the cut, so by convention the cut side in the direction of node numbering advancement around the TF loop is used herein. Maximum shear stress in a section through elastic material is directly proportional to the internal torque state, so this quantity is of special interest. The complete torsion model predicts the distribution of internal torque along with associated small rotational deformations. That necessarily includes the structural support torques and the resulting torsional shear stresses. The model includes, in addition to the distributed current-dependent Lorenz torque loads, the various torsion spring stiffnesses and interconnections that are relevant to rotational deformations.

Since force and torque equilibrium necessarily exist throughout the entire TF system, it follows that the difference in internal torque states between any two sections through the TF conductors is equal to the total external torque acting on the TF conductor region between them. Between the upper and lower leads of the TF centerstack there are no connections able to mechanically transmit torque, so within this region the profile of internal torque state vs location is equal to an additive torque constant plus the electromagnetic torque loading distribution described earlier in this memo. The additive torque constant depends on torsion spring responses of the entire TF conductor and support system, while the electromagnetic torque loading distribution is a known function of currents and position only, independent of any torsion spring responses.

The model approach taken employs the toroidal membrane methods advanced in section VII of Peter Titus' 1999 paper, but extends it in certain ways. For the centerstack portion of the TF system, torsion formulae for thick-walled tubular shafts are used instead of thin-walled approximations. More significantly, discrete mechanical springs augmenting the toroidal membrane model are used to represent effects on the TF conductor system of additional external mechanical support structures and interconnections. These springs include upper and lower umbrella lids together with their ties to the upper and lower umbrella structures, the upper and support rod connections between TF conductor outer legs and the vacuum vessel, the vacuum vessel itself, and the mechanical connections between the vacuum vessel through its gravity support legs to the floor and up through the pedestal pad supporting the TF centerstack. These additional modeled springs are interconnected with each other outside the modeled toroidal membrane and are attached to the toroidal membrane at specific poloidal nodes. Important node numbers in the toroidal membrane model are listed in Table 3 and their locations are depicted in Fig.17.

Table 3: Important Nodes in Toroidal Membrane Model of TF System Conductors

Node Number	R (m)	Z (m)	Importance
1	0.1942	0	Starting node in centerstack's middle
363	0.1942	-2.6000	Bottom of centerstack's straight section
371	0.2450	-2.6067	Bolts Attaching Stepped G10 Ring to Lower SplineLid
383	0.3312	-2.6067	End of Toroidally Continuous TF Lead Conductors
513	0.6923	-2.4916	TF Outer Leg's Lower End
566	1.0489	-2.5165	TF Outer Leg Clamped to Lower Umbrella Structure
838	2.3411	-1.1489	TF Outer Leg Clamp to Rods Connecting to Vacuum Vessel
1163	2.3411	1.1489	TF Outer Leg Clamp to Rods Connecting to Vacuum Vessel
1435	1.0489	2.5165	TF Outer Leg Clamped to Upper Umbrella Structure
1488	0.6923	2.4916	TF Outer Leg's Upper End
1618	0.3312	2.6067	End of Toroidally Continuous TF Lead Conductors
1630	0.2450	2.6067	Bolts Attaching Stepped G10 Ring to Upper SplineLid
1638	0.1942	2.6000	Top of centerstack's straight section
2000	0.1942	0	Ending node in centerstack's middle

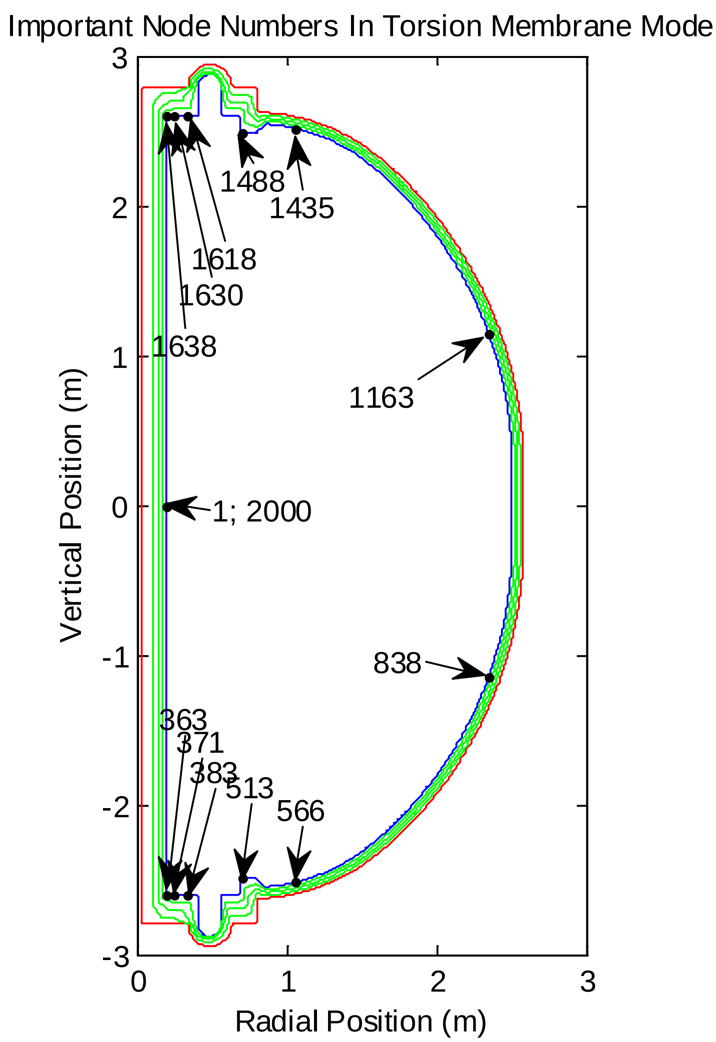


Figure 17: Important Node Numbers in the TF Conductor's Toroidal Membrane Model

The constant-radius portion of the centerstack TF is represented by node numbers 1 through 363 which extend vertically from the centerstack's middle to its bottom of its constant-radius part, and by node numbers 1630 through 2000 which extend vertically from the top of the centerstack's constant-radius part to its middle, where it started at node 1. The centerstack's lower TF lead extensions are node numbers 364 through 383 and its upper TF lead extensions are node numbers 1618 through 1637. The remaining nodes, 384 through 1617, represent the TF outer leg, the TF radial conductors, and the TF flex loops which connect to the centerstack's lead extensions. Electrical connections between the lower and upper ends of each TF outer leg and the lower and upper TF radials are located respectively at node numbers 513 and 1488, each at r=0.6923 meters.

External interconnection paths modeled via discrete torsional springs include the following:

(1) From the upper TF outer legs at node 1435 clamped by a split aluminum block to the vacuum vessel's upper umbrella, to the upper umbrella's bolted lid, radially inwards though the lid to an upper inner bolt plate, then to an annular "crown" disk insulator locked by tight-fitting radial pins to the centerstack's upper leads at node 1630.
(2) The vertical mirror image of (1) on the bottom, i.e., from the lower TF outer legs clamped to the vacuum vessel's lower umbrella at node 566, to its bolted lid, radially inwards to the lower inner bolt plate and insulating "crown" annulus, then to the centerstack's lower leads at node 371.
(3) From the TF outer legs just above horizontal mid-plane ports at node 1163, through rod connections to clevises mounted on the vacuum vessel. The vacuum vessel's upper dome continues this path to the upper umbrella legs then to TF conductor node 1435.
(4) The vertical mirror image of (3) below the midplane, i.e., from the outer legs just below mid-plane ports at node 838, through connecting rods to aa clevis connected to the vacuum vessel, then through the vesse's lower dome, umbrella legs and Aluminum block clamp to node 566.
(5) From node 838 through lower clevis rods, through the vacuum vessel's cylindrical middle section to upper radius rods connections to node 1163.
(6) From the lower vacuum vessel through its mounting legs to the floor, then up through the centerstack's lower pedestal pad support, meeting the centerstack at node 363.

Of primary interest in the investigations for which this torsion model may be applied is how shear stress is distributed within the CenterStack (i.e., the central bundle of turns) portion of the TF system, i.e., in the TF centerstack, since the interturn insulation there must withstand peak values of this stress. Although this is primarily affected by the electromagnetic Lorenz torques directly acting on the TF centerstack, it is also partly dependent on the structures outside the centerstack since mechanical connections with the TF centerstack at its top and bottom allow mechanical transfer of torques there.

TORSIONAL SPRING STIFFNESS FORMULAE

For toroidally continuous portions of a TF conductor system resembling a thin membrane, it is appropriate to model the local rotational deflection due to torsion as follows:

$$\frac{d\phi}{ds} = \frac{T}{2\pi r^3 tG} \qquad (3)$$

where

s represents the independent variable representing poloidal location in the toroidal membrane as a curvilinear distance measured in the membrane's poloidal (r,z) halfplane projection from an arbitrarily chosen reference location on the membrane,

$\phi(s)$ represents the azimuthal (i.e., toroidal) rotation angle at poloidal location s due to torsion,

$\frac{d\phi}{ds}$ is the density of that torsional rotation deflection at location s, that is, the rate of deflection angle increase per unit change in poloidal location,

$T(s)$ is the local internal torque state in the TF membrane conductor at poloidal location s,

G , is the shear modulus of the membrane material,

r(s), is the radius from the z axis of the membrane at poloidal location s, and

t , that is, *t(s),* is the membrane's thickness at poloidal location s.

Relating s to r allows Eq.(1) to be integrated, e.g., if a section of the toroidal membrane is planar so that ds in Eq.(1) equals dr, then integration over the resulting disk-annulus between inner and outer radii yields the following:

$$\phi = \frac{T}{4\pi tG}\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right) \qquad (3a)$$

Another commonly occurring situation has the toroidal membrane in a non-planar configuration. For those cases a small axisymmetric portion of the membrane can be approximated as having a straight line segment for its poloidal projection, extending from (r_1,z_1) to (r_2,z_2). For example, this would apply to a thin approximately conical section taken though a vacuum vessel dome. Integrating over the resulting frustrum of a cone results in the following:

$$\phi = \left(\frac{\sqrt{(r_2 - r_1)^2 + (z_2 - z_1)^2}}{r_2 - r_1}\right)\frac{T}{4\pi tG}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) \qquad (3b)$$

In such portions of the TF conductor system where the thin toroidal membrane model is an appropriate approximation, the shear stress is constant over the thickness of each perpendicular section and varies according to the following formula:

$$\tau = Gr\frac{d\phi}{ds} \quad (4)$$

where $\tau(s)$ the shear stress in the perpendicular section at location s.

In the NSTX CS upgrade's TF system the centerstack bundle of TF turns is sufficiently thick that it should not be modeled as a thin membrane. It is more appropriate to instead model it as an elastic thick-walled tubular shaft in which shear stress in a section perpendicular to the axis varies linearly with radius within that section. For the centerstack portion of the TF conductor system where the inner and outer radii of the TF are constant, the following formulae are used:

$$\frac{d\phi}{ds} = \frac{T}{JG} \quad (5)$$

where J is the rotational moment of inertia of the centerstack TF cross section,

$$J = \frac{\pi}{2}\left(r_{OUTER}^4 - r_{INNER}^4\right) \quad (6)$$

In this thick tubular shaft model the maximum shear stress in each section occurs at its outer radius and is given by the following:

$$\tau_{MAX} = Gr_{OUTER}\frac{d\phi}{ds} \equiv \frac{Tr_{OUTER}}{J} \quad (7)$$

These formulae for thick-walled tubular shafts are also used for lead extension regions of the TF centerstack which extend from r=0.1942 meters out to r=0.3329 meters where toroidal continuity of the TF conductor's mechanical configuration ends. In this 0.1942<r<0.3329 lead extension region the torsion formulae are less accurate since stress concentrations where radii abruptly change are not modeled.

For the present toroidal membrane model of the TF conductor, the blue contour of Figs. 1 through 3 is used for all r>0.3329 outer locations where the thin membrane representation is used. For r<0.3329 locations in the centerstack and its lead extensions where the thick-walled tubular shaft model is more appropriate, the blue contour is used to determine the r_{OUTER} value and the corresponding poloidal point on the red contour is used for r_{INNER}. Together these are used to calculate the local rotational moment of inertia, J, and also the peak shear stress.

Torsional stiffness formulae based on simple first-principles mechanical models are accurate wherever the TF system is nominally continuous in the toroidal direction, which for the NSTX CSU includes all parts of the TF system at locations inboard from the flex loops where the radius is less than 0.3329 meters. However, the same torsion formulae are also used herein to approximately model TF conductor deformation in outer regions where r>0.3329. The TF conductors and their supporting structures are not toroidally continuous in outer regions but instead are broken up by air gaps. The air gaps permit structures to deform in nonaxisymmetric ways not modeled by torsional stiffness formulae. Thus, the present approximation neglecting toroidal air gaps must therefore artificially reduce the stiffness of its assumed toroidally continuous model in order to match its predictions to the actual nonaxisymmetric system. Estimates and other information supplied by Mark Smith and by Tom Willard have been used to adjust these model parameter values.

ESTIMATION OF PARAMETER VALUES

The following section documents both my own ball-park estimates of some stiffness parameters and also torsional stiffness estimates developed by Mark Smith using a global 3D model, which I have preferentially adopted.

Centerstack:

The TF centerstack's new inner radius is 0.0512 m, and its new outer radius is 0.194238 m, not including its lead extensions. The shear modulus of its copper is taken as G=48 GPa, i.e., 4.8e10 Pa. Applying Eq.(3), its twist angle per unit length per unit torque is as follows:

$$\frac{1}{T}\frac{d\phi}{ds} = \frac{1}{JG} = \frac{1}{\frac{\pi}{2}\left(r_{OUTER}^4 - r_{INNER}^4\right)G} = \frac{2}{\pi\left((0.194238)^4 - (0.0512)^4\right)(4.8E10)} =$$

$$= 9.36275866886577\text{e-}009 \text{ radian } \mathrm{N}^{-1} m^{-2} \tag{8}$$

The total centerstack straight section length from node 1 through 363 and 1638 through 2000 is 5.200 m, so its torsional stiffness parameter is

K_{CS}=1/[(9.3627587E-9 radian/N-m^2)(5.200 m)] = 2.054E7 N-m/radian =
= 3.176E6 inch-lbf/degree (9a)

Its reciprocal stiffness parameter is then
R_{CS}= 1/(2.054E7 N-m/radian) =4.85853E-8 radians/N-m. (9b)

Its reciprocal stiffness parameter per element is as follows:
$R_{CS/elt}$= (4.85853E-8 radians/N-m)/(724 elts) = 6.711e-11radians/N-m/elt (9c)

TF Centerstack Lead Extensions

The same formula is used for the lead extension portions of the TF centerstack, nodes 363 through 383 and 1618 through 1638. Its inner radius is also taken as 0.0512 m, but its outer radius varies. Summing the resulting individual reciprocal stiffness terms (using MATLAB) results in

R_{Leads}= 5.835E-10 radians/N-m (10a) for upper lead extensions and the same again result separately for the lower lead extensions.

The average value per element of this constant is as follows:
$R_{Leads/elt}$= (5.835E-10 radians/N-m)/(19 elts) = 3.071E-11 radians/N-m/elt (10b)

The equivalent spring stiffness constant for the lead extensions is the reciprocal, i.e.,

K_{Leads}=1/(5.835E-010 radians/N-m= 1.71E9 N-m/radian (10c)

The upper lead extension is included with the straight part of the centerstack, so
r1=5.8835e-010+4.85853E-8= 4.95e-8 radians per newton-meter (10d)

Insulating Annular "Crown" pinned to leads, bolting plate

Recent design information as of September 2011 is that the upper and lower Crpwn assemblies are axisymmetric structures machined from G10, with each taking the shape of two axially adjacent coaxial rings. Estimated ring dimensions are as follows:

Table 4: Upper Crown Assembly Dimensions (inches)

	Ring 1	**Ring 2**
Inner Diameter	**16.0"**	**17.6"**
Outer Diameter	**21.5"**	**19.4"**
Height	**4.0"**	**2.0"**

Table 5: Lower Crown Assembly Dimensions (inches)

	Ring 1	**Ring 2**
Inner Diameter	**16.0"**	**20.1"**
Outer Diameter	**27.0"**	**23.0"**
Height	**4.0"**	**2.0"**

Assuming the shear modulus of G10 material is 7,69E9 Pa, this results in upper and lower reciprocal stiffnesses (i.e., compliences) of **respectively 5.60E-9 and 2.05E-9 radians/N-m**. (Note that Ref.4 had assumed 5E-9 for both of these.)

Outer Umbrella and Lid:

I had earlier estimated a reciprocal stiffness for the portion of the umbrella between TF clamps and lid as follows:

$$R_{OuterUmbrella}=\left(\frac{1}{T}\frac{d\phi}{ds}\right)(\Delta s)=\left(\frac{\Delta s}{2\pi r^3 t}\right)=\left(\frac{(0.35542)}{2\pi(1.0477521)^3(0.0254)(7.72E10)}\right)=$$

=2.508E-11 radians/N-m. ((11a)

At that time the lid design was envisioned as a stainless steel disk-annulus, assumed to be 1" thick. Its outer and inner radii are assumed to be respectively 1.047752 m and 15.375"/2= 0.1953 m, which resulted in a lid reciprocal stiffness of

$$R=\frac{\phi}{T}=\frac{1}{4\pi tG}\left(\frac{1}{r_i^2}-\frac{1}{r_o^2}\right)=\frac{1}{4\pi(0.0254)(7.72E10)}\left(\frac{1}{(0.1953)^2}-\frac{1}{(1.0477521)^2}\right)=$$

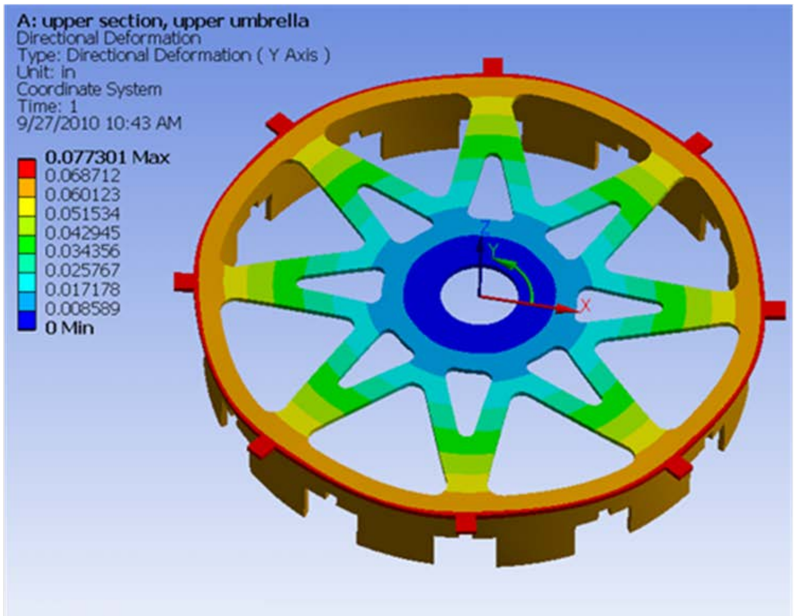
=1.02702e-009 radians per Newton-meter (11b)

Using those values as ballpark estimates, then the net reciprocal stiffness from TF coil clamp to crown interface would have been

2.508e011+1.02702e-009 = 1.052E-9 radian/N-m (11c)

A better estimate of the stiffness via this path was provided by Mark Smith by fitting a global 3D model, as copied into the following text box. The reciprocal stiffness derived from his data is

$R_{umbrella+lid}$=3.338E-9 radians/N-m (11d)



12*10000 lbf = 120000 lbf

R = 41.3 inch

T = 4.956E6 lbf-in

Θ = s/r => .077301 inch/41.3 inch = 0.001872 radians

K1 = T/Θ = 4.956E6 lbf-in/0.001872 radians

K1= 2.64787E9 lbf-in/radians

K1 = 4.6214E7 lbf-in/°

Mark Smith's Global Model Data 1

Inner Umbrella

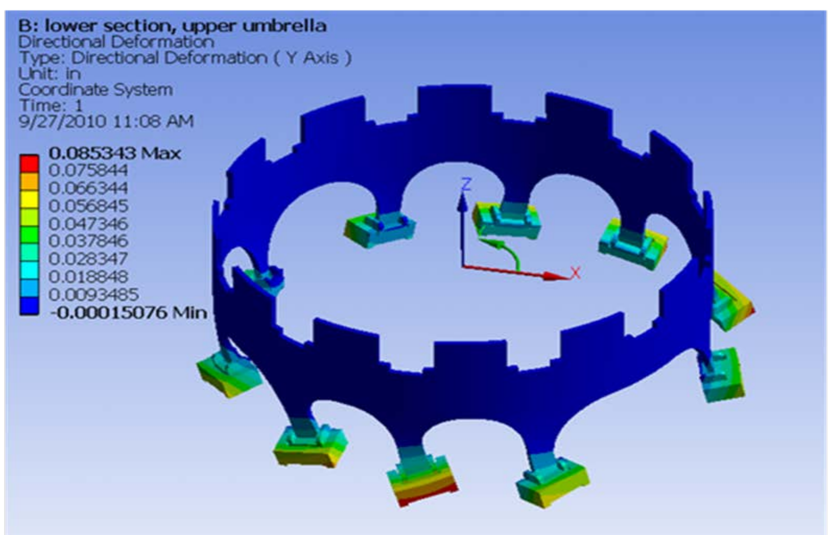
Each umbrella is a nominally cylindrical stainless steel (G=77.2 GPa) structure of radius r=1.0477521 m, thickness t=1"=0.0254 m, and height from base to the middle of the cutouts for TF outer legs is 32.12"=0.8158 m. Eq.(3) could be used to estimate its stiffness parameter if it had no cutouts. However, it has not only the cutouts for TF outer legs but also ten arch-shaped cutouts near its base. If its thickness were reduced by 67% from 1" to t=0.33"=0.00847 m as a ballpark "guestimate" to match actual nonaxisymmetric deflections under torsion loading .the stiffness parameter would become the following:

$$R_{\text{InnerUmbrella}} = \left(\frac{1}{T}\frac{d\phi}{ds}\right)(\Delta s) = \left(\frac{\Delta s}{2\pi r^3 t}\right) = \left(\frac{(0.8158)}{2\pi(1.0477521)^3(0.00847)(7.72E10)}\right) =$$

$$=1.727\text{E-}010 \text{ radians per newton-meter} \qquad (12a)$$

A better estimate of the stiffness of this path was provided by Mark Smith by fitting a global 3D model, as copied into the following text box. It is equivalent to

$$R_{\text{InnerUmbrela}}=4.44\text{E-}9 \text{ radian/N-m} \qquad (12b)$$



M = 4.13E6 lbf-in

R = 41.3 inch

Θ = s/r => .085343 inch/41.3 inch = 0.002066 radians

K2a = T/Θ = 4.13E6 lbf-in/0.002066 radians

K2a=1.99E9 lbf-in/radians

K2a = 3.48827E7 lbf-in/°

Mark Smith's Global Model Data2

Vacuum Vessel Division

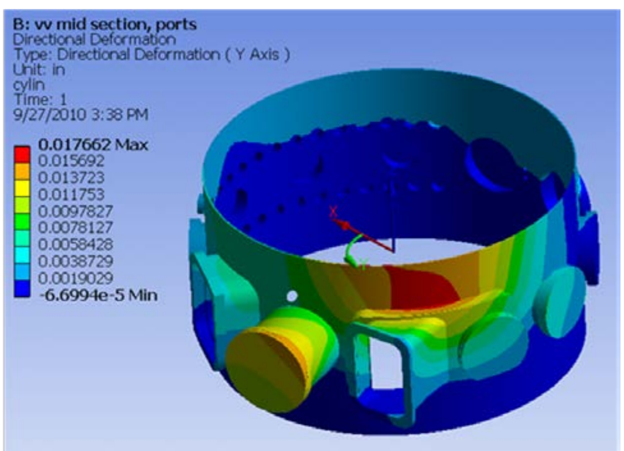
Connecctions to the TF outer legs above and below the horizontal midplane were identified as TF conductor node numbers 838 and 1163, located at (r,z)=(2.3411, ±1.1489) m. These connections go to the vacuum vessel, meeting it at locations closer to the midplane. For the present analysis they are assumed to meet the vacuum vessel at the interface between the vacuum vessel's cylindrical midsection and its upper or lower domes, i.e., at (r,z)=(1.74, 1.00) m. Therefore, for the calculation of spring stiffnesses, one value is for the cylindrical central portion of the vacuum vessel and another is for a vacuum vessel dome connected in series with the portion of the umbrella between the vacuum vessel and the TF outer leg's aluminum block .

Central cylindrical section of the vacuum vessel:

The vacuum vessel is made of stainless steel (shear modulus G=77.2 GPa) of thickness t=0.625"= t=0.01875 m. The radius of the vacuum vessel's central section is about 1.705 m and its height is 2.00 m. Therefore, Eq.(3) could be used for a rough reciprocal stiffness estimate, $R_{VV\text{-}middle}$=4.4367E-11 radians/ N-m. For the effect of port cutouts it was decided to reduce the assumed cylindrical membrane thickness by 50%, to t=0.009375. This would increase the reciprocal stiffness parameter to $R_{VV\text{-}middle}$=8.87E-11 radians/ N-m.

A better estimate of the stiffness of this path was provided by Mark Smith by fitting a global 3D model, as copied into the following text box. It is equivalent to

$$R_{VV\text{-}middle}=5.634\text{-}10 \text{ radian/N-m} \quad (13)$$



M = 4.13E6 lbf-in

R = 67.1 inch

Θ = s/r => 0.017662 inch/67.1 inch = 0.000263 radians

K3 = T/Θ = 4.13E6 lbf-in/0.000263 radians
K3=1.569e10 lbf-in/radians

K3 = 2.738e8 lbf-in/°

Mark Smith's Global Model Data 3

Vacuum vessel dome

Vacuum vessel drawings identified (r.z) coordinates of the upper dome from the dome's mouth to its interface with the vacuum vessel's cylindrical section, as listed in the following table.

Table6: Vacuum Vessel Dome Coordinates

Point #	r (m)	z (m)
1	0.6121	1.7500
2	0.7442	1.7170
3	1.1455	1.5545
4	1.5392	1.3106
5	1.6612	1.1735
6	1.7068	1.0033

The total rotation angle per unit torque for the five unbroken conical frustrum regions bounded by the points is then estimated as follows:

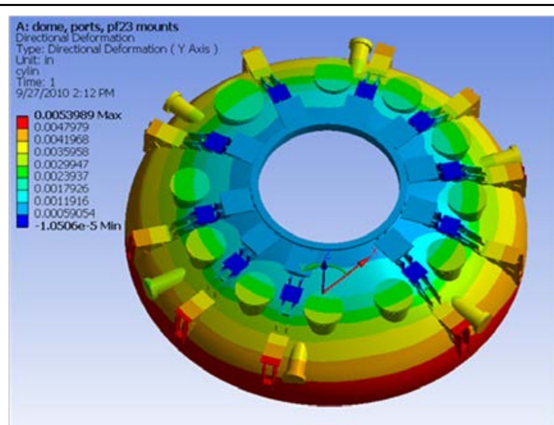
$$R_{dome} = \frac{\phi}{T} = \frac{1}{4\pi t G} \sum_{i=1}^{i=5} \left(\frac{\sqrt{(r_{i+1} - r_i)^2 + (z_{i+1} - z_i)^2}}{r_{i+1} - r_i} \right) \left(\frac{1}{r_i^2} - \frac{1}{r_{i+1}^2} \right) = \frac{1}{4\pi (0.01875)(7.72E10)} (2.57940) =$$

= 1.41805e-010 radians per newton-meter. If one arbitrarily reduces the thickness by 50% to account for port cutouts, this leads to the following guestimate for the reciprocal stiffness of each of the domes:

R_{dome}=2.8361E-10 radian/ N-m (14a)

A better estimate of the stiffness of this path was provided by Mark Smith by fitting a global 3D model, as copied into the following text box. It is equivalent to

R_{dome}=1.72E-10 radian/ N-m (14b)



M = 4.13E6 lbf-in

R = 67.1 inch

Θ = s/r => 0.005399 inch/67.1 inch = 0.00008 radians

K2b = T/Θ = 4.13E6 lbf-in/0.00008 radians
K2b= 5.133e10 lbf-in/radians

K2b = 8.96e8 lbf-in/°

Mark Smith's Global Model Data 4

<u>Vacuum Vessel Connections to TF Outer Legs</u>

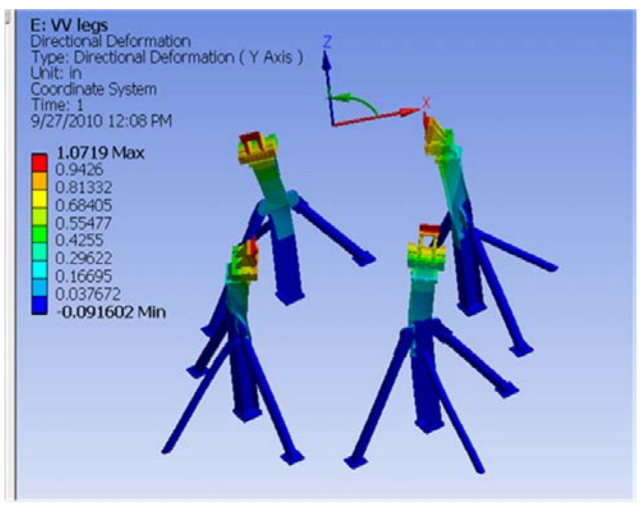
A rough stiffness estimate was made based on assumed 1.5"=0.0381 m diameter solid steel cylindrical rod attached to the vacuum vessel through pin connections at clevis attachments midway between TF outer legs. Coordinates of these connection points are (R,Z)=(1.7068, ±1.00) m, while the coordinates of the TF out legs where they attach to the tangential rods are (R,Z)=(2.3411, ±1.1489) m. Taking into account the 12-fold symmetry and the geometric angles, the resulting reciprocal stiffness parameter estimate is as follows:

$$R_{ClevisRods}=1.22e\text{-}010 \text{ radian/N-m} \tag{15}$$

<u>Vacuum Vessel Legs and Pedestal Support for TF Centerstack</u>

Without nonaxisymmetric analyses it is not possible to estimate the rotational stiffness of the torque path through vacuum vessel support legs, floor, and TF centerstack pedestal support. However, Mark Smith has provided such estimates by fitting a nonaxisymmetric global 3D model, as copied into the following two text boxes. The equivalent reciprocal stiffness parameter for the VV support legs is

$$R_{Legs}=3.72E\text{-}8 \text{ radian/N-m} \tag{16}$$



$M = 4.13E6$ lbf-in

$R = 67.1$ inch

$\Theta = s/r \Rightarrow 1.0719 \text{ inch}/61.7 \text{ inch} = 0.017373$ radians

$K_{legs} = T/\Theta = 4.13E6 \text{ lbf-in}/0.017373$ radians

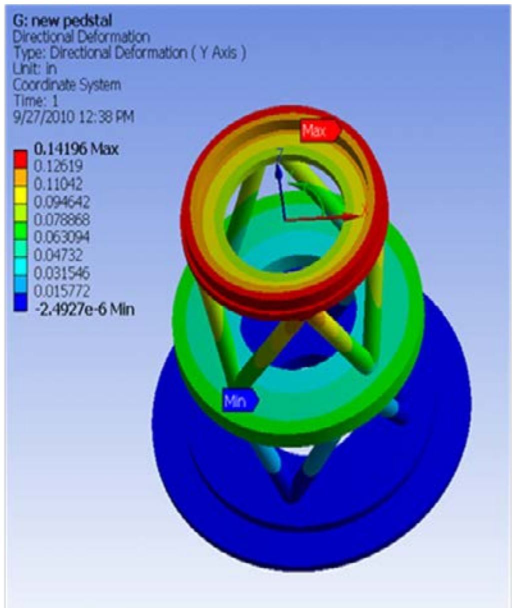
$K_{legs} = 2.37728e8$ lbf-in/radians

$K_{legs} = 4.14914e6$ lbf-in/°

Mark Smith's Global Model Data 5

The equivalent reciprocal stiffness parameter for the pedestal pad supporting the centerstack is:

$$R_{Ped}=2.17E\text{-}8 \text{ radian/N-m} \tag{17}$$



M = 4.13E6 lbf-in

R = 14 inch

Θ = s/r => 0.14196 inch/14 inch = 0.01014 radians

$K_{p\,new}$ = T/Θ = 4.13E6 lbf-in/0.01014 radians

$K_{p\,new}$ = 4.073e8 lbf-in/radians

$K_{p\,new}$ = 7.109e6 lbf-in/°

Mark Smith's Global Model Data 6

<u>TF Conductor Portions Lacking Structural Continuity in Toroidal Direction</u>

The remaining portions of the system are TF conductors which are interspersed with toroidal air gaps lacking any structural rigidity. This situation exists everywhere that the TF conductors have major radius locations exceeding the threshold value, i.e., for R>0.3329 m. Although at certain locations the TF conductors are clamped to other structures, such clamping is not the issue here since those other structures and their TF conductor attachment points are expressly modeled as external springs. Thus, any torsional stiffness parameters to be assigned to toroidally discontiuous portions of TF conductors must be chosen to match the TF conductors' nonaxisymmetric beam-like toroidal deflection behaviors to the axisymmetric model.

Recall that for a straight beam, the deflection distance, *y*, is governed by the following fourth order system of differential equations:

$$\frac{dV}{ds} = q(s)$$

$$\frac{dM}{ds} = V(s)$$

$$\frac{d\theta}{ds} = \frac{M(s)}{EI}$$

$$\frac{dy_f}{ds} = \theta(s)$$

$$\frac{dy_s}{ds} = \frac{V(s)}{AG}$$

$$y = y_s + y_f \tag{18a}$$

where

s is the distance along the straight beam,

q is the perpendicular running load force per unit distance,

V is the internal section shear force,

M is the internal section bending moment,

q is the deflection angle

y_f is the deflection distance due to flexure

y_s is the deflection distance due to shear

E is the elastic modulus

G is the shear modulus

I is the cross section's moment of inertia with respect to the loading direction

A is the cross section's area

Note that the torsion system of equations matches the shear part of the beam deflection system but has no way of modeling beam flexure. Thus, there is no first-principles approach to matching the models. As an intuitive example of this mismatch, a continuous toroidal membrane representing the TF outer legs would develop nonzero internal torque if its top were rotated by, say, 0.001 radians with respect to its bottom, but the collection of 12 TF outer legs could sustain such a relative rotation without elastically deforming them (to first order), i.e., with twelve different small rigid body tilting motions of the separate outer legs.

Thus, the best that can be done is to make a very approximate, order-of-magnitude torsional stiffness estimate for the outer TF conductor, as a "kludge". To this end, Tom Willard supplied ANSYS calculated results for the out-of-plane deflection of an isolated TF Outer Leg under defined loading conditions. The model imposed cantilever restraints on the conductor at the upper and lower locations where the actual outer leg is clamped to the umbrellas by aluminum blocks (i.e., the torsion model's nodes 566 and 1435) It applied 1000 lbf=4454.5 N out-of-plane load forces to the outer legs at the locations where each acual outer leg will be connected to vacuum vessel clevises via rods (i.e., the torsion model's nodes 838 and 1163). Maximum calculated OOP deflection was 0.073 inches with a single OOP force applied and 0.140 inches=3.56 mm with both OOP forces applied in the same direction. The torsion model's stiffness value is then set by the second of these results, based on the deflection of the top half only. A torque of

$$T=(12 \text{ outer legs})(F=4454.5 \text{ N})(R=2.3411 \text{ m}) = 125{,}142 \text{ N-m}$$

is predicted by ANSYS to cause the TF conductor portion between the upper umbrella's clamp (node 1435) and the clevis rod clamp (node 1163) to rotate through and angle of

$$\Delta\phi= (0.00356 \text{ m})/(2.3411 \text{ m}) = 0.0015189 \text{ radians}$$

The corresponding reciprocal stiffness value is then

$$R_{1163:1435} = \Delta\phi /T = (0.0015189 \text{ radians})/(125{,}142 \text{ N-m}) = \\ =1.2138\text{e-}008 \text{ radian/N-m} \qquad (18b)$$

In order to assign a consistent stiffness to other portions of the TF outer legs, a single-element value is first obtained by dividing the above result by the 1435-1163+1= 273 elements to obtain

$$R_{TFOL/elt}=R_{1163:1435} /(1435-1163+1 \text{ elts}) = 4.45\text{E-}11 \text{ radian/N-m/elt} \qquad (18c)$$

This per element value is then used to generate reciprocal stiffness values for the other portions of the TF outer legs. For the portion between lower and upper rod attachments to clevises,

$$R_{838:1163}=(1163-838+1)R_{TFOL\text{-}node} = 1.45\text{E-}8 \text{ radian/N-m} \qquad (18d)$$

For the Outer leg TF portion inboard of each umbrella clamp, including the flex straps for which no better "kludge" estimate of torsional stiffness has been derived, the reciprocal stiffness values are as follows:

$$R_{1435:1618}= R_{383:566}== (184)R_{TFOL\text{-}node} = 5.26\text{E-}8 \text{ radian/N-m} \qquad (18e)$$

THE EQUIVALENT ELECTRICAL MODEL

This analysis uses an analogy between the torsional mechanical system and a hypothetical electrical network. The angular twist of an axisymmetic portion of the TF system is analogous to the voltage developed across a length of resistive conductor in the hypothetical analogous electrical network, and the internal torque developed in that axisymmetric section is analogous to the current flowing through the analogous resistor. Torsional spring stiffnesses, expressed in units of torque per unit angular deflection, are analogous to conductances (i.e., reciprocal resistances) in the hypothetical electrical network.

Since electromagnetic torque loading on the TF centerstack is not spatially umiform, the TF centerstack can best be modeled as a series-connected sequence of many small torsion springs. The nodes between the resistors represent a sequence of physical locations within the centerstack. The nonuniform distributed torque loading is then modeled by assigning the Lorenz torque increment for each region between two adjacent nodes to one of those two nodes. (Little error is introduced by this asymmetrical assignment if nodes are closely spaced.) As developed in the Appendix, this torque is equal to the product of total TF threading current, $N_{TF}I_{TF}$, times the difference in poloidal flux per radian between the modeled element's two ends, i.e.,

$$T_j = N_{TF} I_{TF} \frac{\Psi_{j+1} - \Psi_j}{2\pi} \qquad (!9)$$

An electrical analogy shown in Fig.18a is a series connected string of resistors interleaved with current sources that inject increments of electrical current from a common reference node.

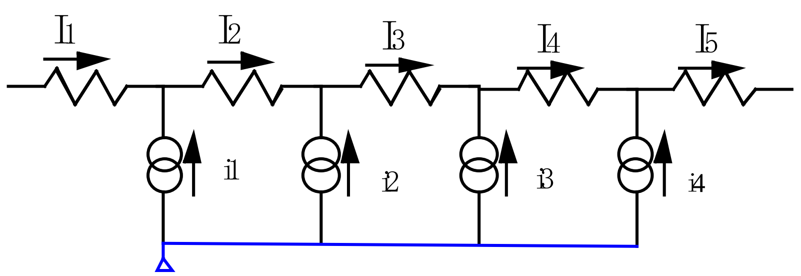


Figure 18a: Electrical Analogy of Series-Connected Torsion Springs And External Torques

Currents I_j flowing through the Fig.18a resistors represent the internal torque state at different vertical locations in the TF centerstack. Resistor currents can thus differ from each other according to:

$$I_{j+1} = I_j + i_j \qquad (20)$$

where i_j is an injected current. Since the injected currents, i_j, represent the incremental torques, T_j, the electrical analogy model's resistor currents are related to actual poloidal fluxes as follows:

$$I_j = N_{TF} I_{TF} \frac{\Psi_j}{2\pi} + C \qquad (21)$$

where C is any fixed constant value.

This Fig18a model can be continued beyond the TF centerstack through the outer parts of the TF conductor system where mechanical supports interconnecting portions of the system must also be modeled. The fact that the total net Lorenz torque on the entire TF system is necessarily zero translates in the full TF system model's analogous electrical network into the net current from the common reference node being zero. This allows the Fig.18b alternative equivalent circuit model to be used instead for the analogy.

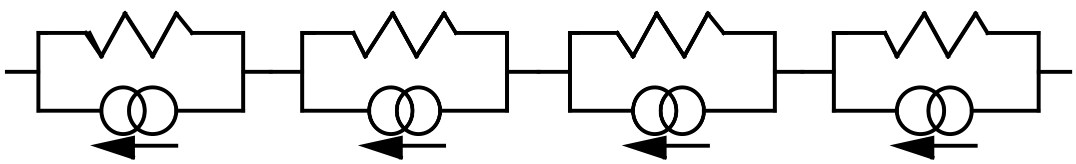


Figure 18b: Alternative Analogy of Series-Connected Torsion Springs And External Torques

In Fig.18b, the current in each external current source is assigned as:

$$i_j = N_{TF} I_{TF} \frac{\Psi_j}{2\pi} \tag{22}$$

where j is the node number of the location at one of the current source's ends. With this assignment, the current in each of the model's resistors is related to actual poloidal flux as

$$I_j = N_{TF} I_{TF} \frac{\Psi_j}{2\pi} + C \,, \tag{23}$$

where C is the common constant value of current flowing between series blocks of Fig.18b. This also clearly matches the resistor current in the Fig.18a model.

Fig.19 depicts the electrical analogy of the full toroidal membrane model of the TF conductor, which includes 1999 copies of the basic blocks of Fig.18b, all connected in series to form a single loop. Fig.19 shows only 25 of the 1999 copies, but the rest are implied.

The current in each Fig.19 primary resistor representing the TF centerstack equals the current in its parallel current source plus the loop current in the ccw direction. Analogously, the internal torque state at any location is the applied differential Lorenz torque plus the torque mechanically transferred in. Other parts of Fig.19 are also interconnected through external resistors lacking any parallel current sources (i.e., lacking any applied EM torques). Figure 19 depicts these external paths with bold symbols, using resistance numberings keyed to the previously numbered external paths for mechanical torques.

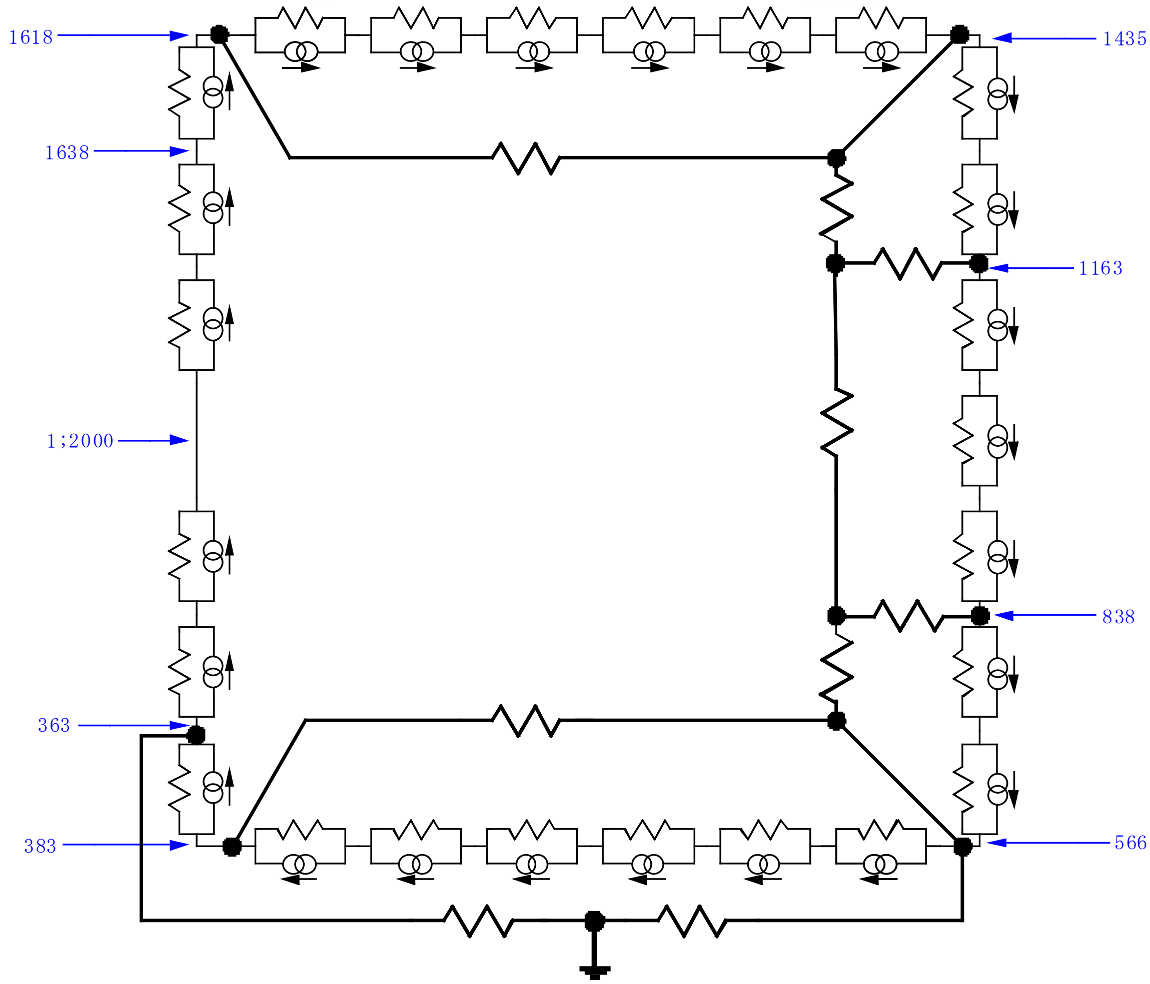


Figure 19: Electrical Analogy TF Torsional Model for NSTX CSU

The external resistors of Fig.19 are identified as follows. R(1) and R(2) represent series mechanical connection paths from respectively upper and lower aluminum block TF conductor clamps through the upper or lower umbrella to the umbrella lid and its radial connection to the insulating crown bolted to the centerstack's TF leads. R(3A) represents the upper system of rods connecting between TF outer leg clamps and clevis attachments to the vacuum vessel. R3B represents the series combination of the upper dome portion of the vacuum vessel above the upper clevis attachments, the upper umbrella mounting, and the upper umbrella up to where aluminum block TF conductor clamps are attached. R(4A) and R(4B) represent the lower mirror image of R(3A) and R(3B). R(5), together with R(3A) and R(4A), implement the previously listed interconnection path (5). R(6A) in series with R(6B) together model path (6) which transmits torque through the vacuum vessel's legs to the floor and then up through the pedestal pad to the centerstack's bottom. In this model, the torque through path (6) is transmitted into the inner part of the centerstack's bottom without going through the lower lead extensions.

The following table summarizes electrical analogy model resistances from the previous section.

Table 7: Electrical Analogy Model Resistance Values

Parameter	Comments	Analogous Resistance Value (radian/N-m)
R(1)	$R_{crown}+R_{umbrella+lid}$	8.938E-9
R(2)	$R_{crown}+R_{umbrella+lid}$	5.388E-9
R(3B)	$R_{InnerUmbrela}+R_{dome}$	4.962E-9
R(4B)	$R_{InnerUmbrela}+R_{dome}$	4.962E-9
R(3A)	$R_{ClevisRods}$	1.22E-10
R(4A)	$R_{ClevisRods}$	1.22E-10
R(5)	$R_{VV\text{-}middle}$	5.634-10
R(6A)	R_{Ped}	2.17E-8
R(6B)	R_{Legs}	3.72E-8
$R_{1638:2000\&1:363}$	R_{CS}	4.86E-8
$R_{363:383}$	R_{Leads}	5.835E-10
$R_{1618:1638}$	R_{Leads}	5.835E-10
$R_{383:566}$		5.26E-8
$R_{1435:1618}$		5.26E-8
$R_{566:838}$		1.214E-8
$R_{1163:1435}$		1.214E-8
$R_{838:1163}$		1.45E-8

SOLUTION OF NSTX CSU ELECTRICAL ANALOGY TF TORSIONAL MODEL

In order to calculate the shear stress profile in the TF centerstack it is necessary to first determine the internal torque state as a function of location, $T(s_j)$. In terms of the Fig.19 electrical analogy model this is equivalent to calculating resistor currents in each of the Fig.18b-style blocks representing the centerstack, i.e., between nodes 1 and 371 and between nodes 1630 and 2000 (a.k.a. 1). However, each such resistor current is simply the sum of the constant value of current passing between all of the centerstack's Fig.18b-type blocks and the known poloidal-flux-dependent current in its parallel current source. Thus, only two constant current values need to be determined to evaluate centerstack shear stress profiles. One current value is needed for nodes 1630 through 2000 and 1 through363, while the other is needed for nodes 363 through 371 which represent the lower lead extensions. Furthermore, inspection of Fig.19 shows that the entire TF system model could be similarly treated by solving for a total of seven (7) such "constant" values of mesh currents.

Determining terminal currents is simplified by equivalent-circuit reductions. A Thevenin equivalent of the basic block in the Fig.18b circuit employing an ideal series voltage source is shown in Fig.20. The voltage of this source is the product of the Fig.18b block's source current and resistance, i.e.,

$$V_j = R_j N_{TF} I_{TF} \frac{\Psi_j}{2\pi} \tag{24}$$

and its resistance is equal to R_j, the same as in the Fig.18b block. It has the same external terminal node voltage/current characteristics as Fig 18b, ***but its internal resistor current does not match the current through Fig.18's resistor***.

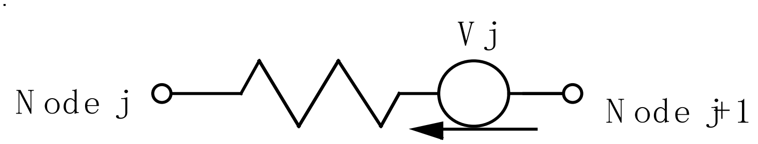


Figure 20: Thevenin Equivalent of the Fig.21 Circuit, Using A Voltage Source

Converting from Fig.18b-type blocks to the Fig.20 form allows many series-connected blocks to be merged into a single block by adding their resistances and adding their source voltages. Applying this substitution to Fig. 19 and combining blocks results in the Fig.21 circuit, which for mesh currents is equivalent to the Fig.19 circuit but easier to calculate. After solving for them, they are substituted into the Fig.19 circuit in order to calculate each resistor current there which is analogous and proportional to the internal torque state of the corresponding part of the mechanical system.

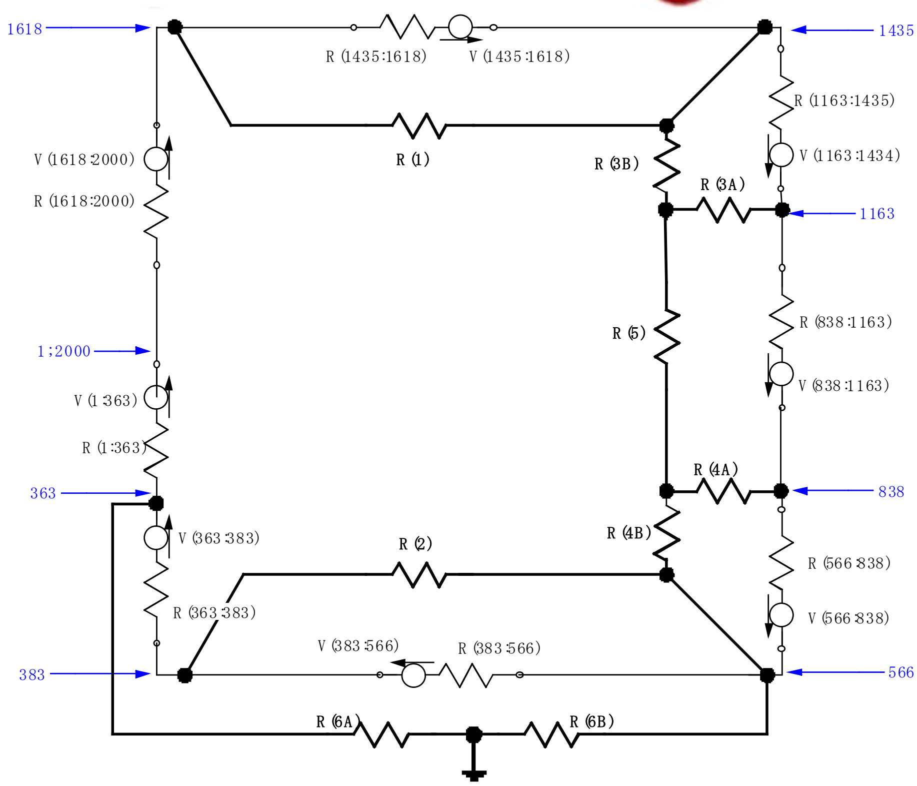


Figure 21: Equivalent Electrical Analogy TF Torsional Model for NSTX CSU

To help solve the Fig.21 circuit it is appropriate to first combine series elements and assign more compact names to designate the resistances, voltage source parameters, and distinct mesh currents in the circuit. This is done in the following diagram which uses 7 mesh loops. Note that the mesh loop carrying the current, i1, is the large loop that contains all seven voltage sources.

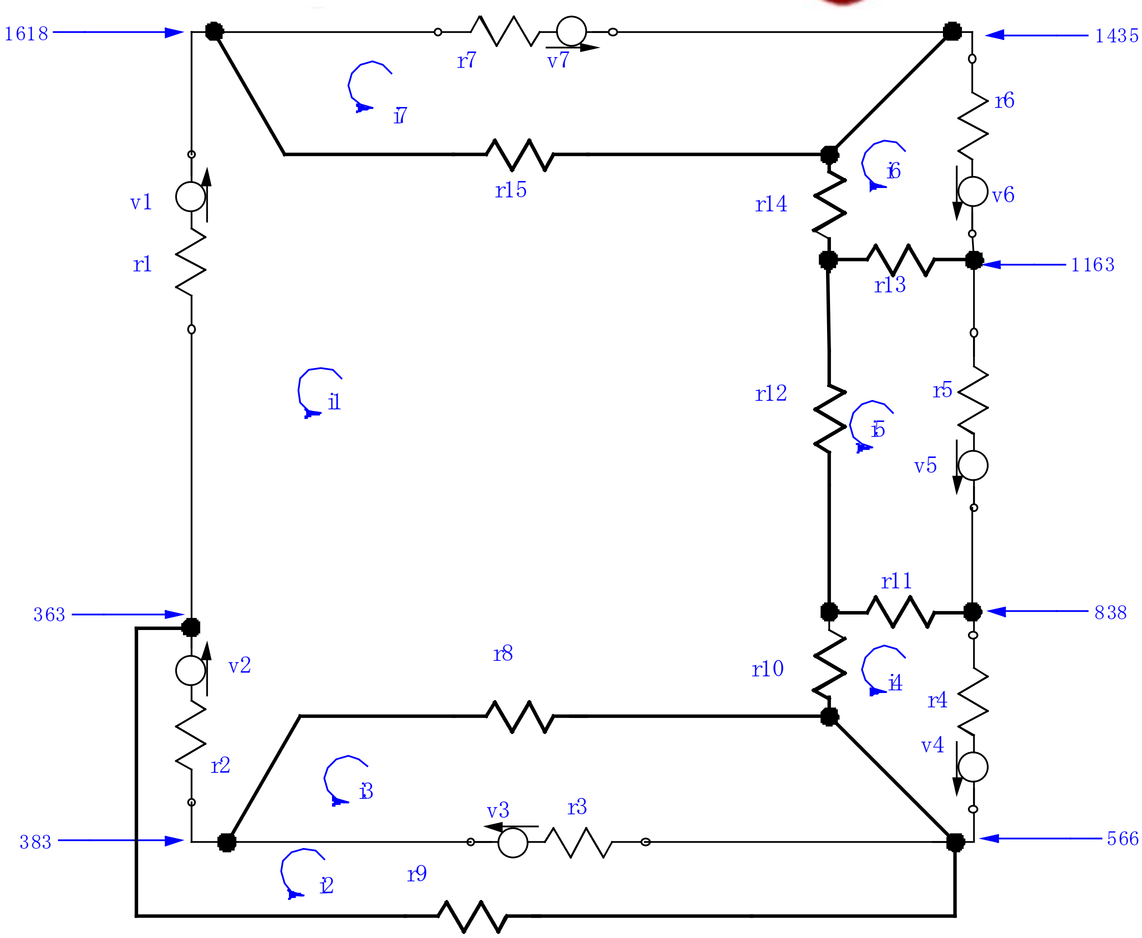


Figure 22: Simplified equivalent circuit of electrical analogy torsion model

The correspondences with symbols of earlier diagrams and parameter values are as follows:

r1=R(1:363)+R(1638:2000) +R(1618:1638)
= 4.86E-8 + 5.8835E-10 = 4.95E-8 radian/N-m

r2=R(363:383)= 5.8835E-10 radian/N-m

r3=R(383:566) = 5.26E-8 radian/N-m

r4=R(566:838)= 1.214E-8 radian/N-m

r5=R(838:1163) = 1.45E-8 radian/N-m

r6=R(1163:1435) = 1.214E-8 radian/N-m

r7=R(1435:1618)= 5.26E-8 radian/N-m

r8=R(2) = 5.388E-9 radian/N-m

r9=R(6A)+R(6B) = 2.17E-8 +3.72E-8 = 5.89E-8 radian/N-m

r10 = R(4B) = 4.61E-9 radian/N-m

r11=R(4A)= 1.22E-10 radian/N-m

r12 = R(5) = 5.634-10 radian/N-m

r13 = R(3A) = 1.22E-10 radian/N-m

r14 = R(3B) = 4.61E-9 radian/N-m

r15 = R(1) = 8.938E-9 radian/N-m

(25a-25o)

The analogous voltage sources result from the series connection of a sequence of elements each containing its own voltage source. When a series sequence of such elements have common torsional "resistance" values which multiply their equivalent current sources, these common resistances can be factored out of the voltage sums yielding a sum over the equivalent sequence of analog current sources. However, since each analog current source is proportional to a difference of successive poloidal flux values, a simplifying cancelation occurs which eliminates poloidal flux at the intermediate nodes results.

In the reciprocal stiffnesses calculated for the TF conductors, constant elemental resistance values result for the constant-radius portions of the TF center-stack. A second constant value per element has been selected for the TF outer leg conductor, extending inwards to the flex straps. The lead extensions were not modeled as having such a constant value but will nevertheless be approximated with their constant average value in order to simplify calculations. The three constant resistance values per element are as follows.

$R_{CS/elt}$= 6.7e-11radians/N-m/elt = R_{CSe}

$R_{Leads/elt}$=3.071E-11 radians/N-m/elt =R_{Le}

$R_{TFOL/elt}$=4.45E-11 radian/N-m/elt =R_{TFOLe} (26)

Then the analogy model's ideal voltage sources can be expressed as follows

$$v1 = \left(R_{\text{CS/elt}} \left(\sum_{i=1638}^{1999} \Psi_i + \sum_{i=1}^{363} \Psi_i \right) + R_{\text{Leads/elt}} \sum_{i=1618}^{1637} \Psi_i \right) \frac{N_{TF} I_{TF}}{2\pi} \tag{27a}$$

$$v2 = \left(R_{\text{Leads/elt}} \sum_{i=364}^{383} \Psi_i \right) \frac{N_{TF} I_{TF}}{2\pi} \tag{27b}$$

$$v3 = \left(R_{\text{TFOL/elt}} \sum_{i=384}^{565} \Psi_i \right) \frac{N_{TF} I_{TF}}{2\pi} \tag{27c}$$

$$v4 = \left(R_{\text{TFOL/elt}} \sum_{i=566}^{837} \Psi_i \right) \frac{N_{TF} I_{TF}}{2\pi} \tag{27d}$$

$$v5 = \left(R_{\text{TFOL/elt}} \sum_{i=838}^{1163} \Psi_i \right) \frac{N_{TF} I_{TF}}{2\pi} \qquad \text{(27e\_}$$

$$v6 = \left(R_{\text{TFOL/elt}} \sum_{i=1164}^{1434} \Psi_i \right) \frac{N_{TF} I_{TF}}{2\pi} \tag{27f}$$

$$v7 = \left(R_{\text{TFOL/elt}} \sum_{i=1435}^{1617} \Psi_i \right) \frac{N_{TF} I_{TF}}{2\pi} \tag{27g}$$

Using these variables the seven mesh equations to be solved for mesh currents are as follows:

$$r1*i1+r2*(i1-i2)+r3*(i1-i2+i3)+r4*(i1+i4)+r5*(i1+i5)+r6*(i1+i6)+r7*(i1+i7)$$
$$=-(v1+v2+v3+v4+v5+v6+v7)$$

$$r9*i2+r3*(i2-i1-i3)+r2*(i2-i1)=v2+v3$$

$$r3*(i3+i1-i2)+r8*i3=-v3$$

$$r4*(i1+i4)+r11*(i4-i5)+r10*i4=-v4$$

$$r5*(i1+i5)+r13*(i5-i6)+r12*i5+r11*(i5-i4)=-v5$$

$$r6*(i1+i6)+r14*i6+r13*(i6-i5)=-v6$$

$$r7*(i1+i7)+r15*i7=-v7$$

(28)

(These equations can be expressed more compactly in vector-matrix form , as follows:

$$\begin{bmatrix} (r1+r2+r3+r4+r5+r6+r7) & -(r2+r3) & r3 & r4 & r5 & r6 & r7 \\ -(r2+r3) & (r2+r3+r9) & -r3 & 0 & 0 & 0 & 0 \\ r3 & -r3 & (r3+r8) & 0 & 0 & 0 & 0 \\ r4 & 0 & 0 & (r11+r4+r10) & -r11 & 0 & 0 \\ r5 & 0 & 0 & -r11 & (r5+r11+r12+r13) & -r13 & 0 \\ r6 & 0 & 0 & 0 & -r13 & (r6+r13+r14) & 0 \\ r7 & 0 & 0 & 0 & 0 & 0 & (r7+r15) \end{bmatrix} \begin{bmatrix} i1 \\ i2 \\ i3 \\ i4 \\ i5 \\ i6 \\ i7 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \end{bmatrix} \tag{29}$$

where

$$\begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & R_{Le} & R_{CSe} \\ R_{Le} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{TFOLe} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{TFOLe} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{TFOLe} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{TFOLe} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{TFOLe} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Psi_{364:383} \\ \Psi_{384:565} \\ \Psi_{566:837} \\ \Psi_{838:1163} \\ \Psi_{1164:1434} \\ \Psi_{1435:1617} \\ \Psi_{1618:1637} \\ \Psi_{1638:1999} + \Psi_{1:363} \end{bmatrix} \left(\frac{N_{TF} I_{TF}}{2\pi} \right) \tag{30}$$

DISCUSSION OF ANALYSIS RESULTS

For the stiffness parameter values chosen herein and for each of 97 coil current combination cases including the 96 plasma equilibria (specified previously by J. Menard) and a single OH-only +24 kA precharge case, the model's solution was obtained via MATLAB. Once the fixed currents i1 and i2 for a parameter case were numerically found the were used to determine the corresponding internal torque and shear stress profile in the TF centerstack. The resulting shear stress profiles are plotted in Appendix 2. The maximum peak absolute shear stress over all 97 cases examined was 25.234 MPa, but many cases had almost this large a value of peak absolute shear stress. Inspection of the profiles shows that the OH coil's effect on peak shear stress is stronger than the combined effects of the other PF coils and the plasma, but that the 25 MPa peak shear only occurs when the OH current magnitude is maximized with the polarity (negative) that reinforces the polarity of shear stress resulting from the equilibrium PF coil and plasma currents alone. In such cases the shear stress at the middle of the inner leg is near zero, indicating that almost all of the torsion load is carried through the vacuum vessel with essentially none is carried through the inner leg.

Stiffness parameters were then perturbed to find their effects on peak shear stress. It was found that reducing the stiffness connecting the TF inner leg to the vacuum vessel reduces the peak shear stress in the inner leg. The stiffness reduction achieves this by reducing the load carried through the vacuum vessel, thus increasing the load share carried through the inner leg. This shifts the inner leg's shear stress profile by a constant amount at all vertical locations, thus decreasing peak torsion in the critical regions near the inner leg's top and bottom while increasing the oppositely directed torsion at the inner leg's middle. Appendix 3 presents results for a case in which the upper lid's stiffness is reduced by 95%, without any other changes. Comparison of stress distribution plots in Appendix 3 with those of Appendix 2 reveals how this stress profile shift causes peak torsion shear stress to be reduced by 22.5%.

APPENDIX 1

Out-Of-Plane (OOP) Torque Algorithm Exposition

Derivation of Torsion Load Formulae for Out-Of-Plane (OOP) forces on Toroidal Field Coils

In general, the total moment (i.e., torque) vector of electromagnetic forces about an origin is the volume integral of the following differential:

$$d\vec{M} = \vec{r} \times \left(\vec{J} \times \vec{B}\right) dV \tag{A1}$$

where $\vec{r}$ is the position vector of a differential volume, dV, $\vec{J}$ is the current density vector and $\vec{B}$ is the magnetic field vector. For a toroidal field coil system it is appropriate to use a cylindrical coordinate system. *(r,θ,z)*, in which the vertically oriented z axis is the central axis of symmetry which includes the origin. To analyze OOP forces in a near-axisymmetric system it is sufficient to consider only current densities and magnetic fields lying within the local poloidal plane and depending only on the position vector within that same plane. Thus, these vectors can be rewritten as follows:

$$\begin{aligned} \vec{r} &\equiv r\hat{r} + z\hat{z} \\ \vec{J} &\equiv J_r\hat{r} + J_z\hat{z} \\ \vec{B} &\equiv B_r\hat{r} + B_z\hat{z} \end{aligned} \tag{A2}$$

where $\hat{r}, \hat{\theta}, \hat{z}$ are unit vectors aligned with the local coordinate system directions.

The volume differential in cylindrical coordinates becomes:

$$dV \equiv r\, drd\theta\, dz \tag{A3}$$

Substituting and combining terms to simplify the result, the differential moment vector is rewritten as follows in Eq. (A4)

$$d\vec{M} = \left(J_z B_r - J_r B_z\right)\left(r\hat{z} + z\hat{r}\right) rdrd\theta dz \tag{A4}$$

When integrating Eq.(A4) over the full range of toroidal angle, $0 \le \theta < 2\pi$, the radial unit vector term, $\hat{r}$, cancels itself out for rotationally symmetric poloidal magnetic fields and TF coil current densities. The remaining nonzero part of the integral is stated in Eq.(A5).

$$\vec{M} = \hat{z}\iiint \left(J_z B_r - J_r B_z\right) r^2 drd\theta dz = \hat{z}\; 2\pi \iint \left(J_z B_r - J_r B_z\right) r^2 drdz \tag{A5}$$

In this last form, the double integral is taken over the (r,z) poloidal half-plane. However, current density components are zero everywhere outside the TF coils so the integration region only needs to include the (r,z) projection of TF coil conductors.

An important simplification results from changing over to stream function variables. For axisymmetric systems the poloidal flux stream function, *Ψ(r,z)*, is the total magnetic flux enclosed by the circle centered on and normal to the z axis which passes through (r,z). Poloidal magnetic flux is related to the poloidal magnetic field as stated by Eq.(A6).

$$B_r(r,z) = -\frac{1}{2\pi r}\frac{d\Psi(r,z)}{dz}$$
$$B_z(r,z) = \frac{1}{2\pi r}\frac{d\Psi(r,z)}{dr} \tag{A6}$$

so

$$\vec{B} = \frac{1}{2\pi r}\hat{\theta} \times \nabla\Psi \tag{A7}$$

We similarly define the toroidal field coil current stream function, *I(r,z)*, as the total TF coil current enclosed by the circle about the z axis passing through (r,z). This current stream function is related to the TF current density as stated by Eq.(A8).

$$J_r(r,z) = -\frac{1}{2\pi r}\frac{dI(r,z)}{dz}$$
$$J_z(r,z) = \frac{1}{2\pi r}\frac{dI(r,z)}{dr} \tag{A8}$$

so

$$\vec{J} = \frac{1}{2\pi r}\hat{\theta} \times \nabla I \tag{A9}$$

Substituting these stream functions of Eqs.(A7) and (A9) into the previous integral yields Eq.(A10a).

$$\begin{aligned} \vec{M} &= \iiint \vec{r} \times \left(\vec{J} \times \vec{B}\right) dV = \\ &= \iiint (r\hat{r} + z\hat{z}) \times \left(\left(\frac{1}{2\pi r}\hat{\theta} \times \nabla I\right) \times \left(\frac{1}{2\pi r}\hat{\theta} \times \nabla\Psi\right)\right) rdrdzd\theta = \\ &= \frac{1}{4\pi^2}\iiint\left(\hat{z} - \frac{z}{r}\hat{r}\right)\left(\hat{\theta} \bullet (\nabla I \times \nabla\Psi)\right) drdzd\theta \end{aligned} \tag{A10a}$$

As stated previously, the radially oriented term cancels out while integrating over toroidal angle, leaving Eq.(A10b) as the result.

$$\vec{M} = \frac{\hat{z}}{2\pi}\iint drdz\left(\frac{\partial I}{\partial r}\frac{\partial\Psi}{\partial z} - \frac{\partial I}{\partial z}\frac{\partial\Psi}{\partial r}\right) = \frac{\hat{z}}{2\pi}\iint\left(\hat{\theta} \bullet (\nabla I \times \nabla\Psi)\right) drdz \tag{A10b}$$

This is a particularly simple and compact formula involving the integral of the cross product of gradients of two scalar functions. The poloidal magnetic flux function can be directly obtained to any desired accuracy by use of Greens functions involving the standard elliptic integral functions, K() and E(), and the current stream function can be approximated using the projected outline of TF conductors. Furthermore, the integral itself can be approximated from these data using very simple algorithms.

Limit for the case of a slender TF conductor

Vector identities applied to Eq.(A10b) imply that

$$\vec{M} = -\frac{\hat{z}}{2\pi}\iint drdz\left(\left(\nabla I \times \hat{\theta}\right)\bullet \nabla\Psi\right) \qquad \text{(A11)}$$

Here, the integration is over the area of the poloidal projection of the TF conductor segment whose net torque is to be calculated. If the conductor's projection is slender with a small width, w, we can change the element of poloidal area from dA=drdz to dA=dldw where l represents distance along the conductor's length. Assuming constant current density in the conductor the gradient vector of the current stream function, ∇I , has a magnitude equal to the total TF urrent divided by the width, $\left|\nabla I\right| = \frac{I_{TF}}{w}$, and the gradient vector 's direction is perpendicular to the local TF current streamline, pointing towards the coil's bore. It follows that $\left(\nabla I \times \hat{\theta}\right)$ has the same magnitude but is pointed in the same direction as the flowing TF current, a direction denoted here by the unit vector, $\hat{n}$. With these substitutions, the net torque over a slender TF conductor segment extending from point A to point B can be rewritten as in Eq.(A12).

$$\begin{aligned} \vec{M} &= -\frac{\hat{z}}{2\pi}\iint drdz\left(\left(\nabla I \times \hat{\theta}\right)\bullet \nabla\Psi\right) \\ &= -\frac{\hat{z}}{2\pi}\frac{I_{TF}}{w}\int_0^w dw\int_A^B dl\ \hat{n}\bullet\nabla\Psi \qquad \text{(A12)} \\ &= -\frac{\hat{z}}{2\pi}I_{TF}\int_A^B dl\ \hat{n}\bullet\nabla\Psi \end{aligned}$$

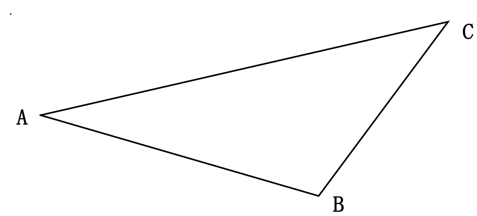
However, this last integral expression in Eq.(A12) may be immediately recognized as the line integral of the gradient of a scalar function, so it has the following Eq.(A13)exact solution:

$$\vec{M} = \hat{z}\ I_{TF}\ \frac{\Psi_A - \Psi_B}{2\pi} \qquad \text{(A13)}$$

Thus, the general formula of Eq.(A12), when interpreted for slender conductors, asserts that the net torque over a poloidal length of TF conductor is simply the product of the difference between the poloidal flux values at the conductor's ends, divided by 2π, then multiplied by the TF current.

Derivation of Approximation For Wider TF Conductors

Consider a triangular region in the poloidal half-plane, ΔABC, on which the exact scalar functions *I(r,z)* and *Ψ(r,z)* are to be linearly approximated by $\tilde{I}(r,z)$ and $\tilde{\Psi}(r,z)$ using function values that are exact at the triangle's three corners. The linear models are as follows:



$$
\begin{aligned}
I(r,z) &\approx \tilde{I}(r,z) = \left(\tilde{I}_0\right) + \left(\frac{\partial \tilde{I}}{\partial r}\right) r + \left(\frac{\partial \tilde{I}}{\partial z}\right) z \\
\Psi(r,z) &\approx \tilde{\Psi}(r,z) = \left(\tilde{\Psi}_0\right) + \left(\frac{\partial \tilde{\Psi}}{\partial r}\right) r + \left(\frac{\partial \tilde{\Psi}}{\partial z}\right) z
\end{aligned}
\tag{A14}
$$

where $\left(\tilde{I}_0\right), \left(\frac{\partial \tilde{I}}{\partial r}\right), \left(\frac{\partial \tilde{I}}{\partial z}\right), \left(\tilde{\Psi}_0\right), \left(\frac{\partial \tilde{\Psi}}{\partial r}\right), \left(\frac{\partial \tilde{\Psi}}{\partial z}\right)$ are linear model coefficient parameters that have constant values throughout the triangle. The requirement to match the approximation to actual function values at triangle corners yields the following matrix equations, where $(r_A, z_A), (r_B, z_B), (r_C, z_C)$ are the coordinates of the triangle's corners:

$$
\begin{bmatrix} 1 & r_A & z_A \\ 1 & r_B & z_B \\ 1 & r_C & z_C \end{bmatrix}
\begin{bmatrix} \left(\tilde{I}_0\right) \\ \left(\frac{\partial \tilde{I}}{\partial r}\right) \\ \left(\frac{\partial \tilde{I}}{\partial z}\right) \end{bmatrix}
= \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}
\tag{A15}
$$

$$
\begin{bmatrix} 1 & r_A & z_A \\ 1 & r_B & z_B \\ 1 & r_C & z_C \end{bmatrix}
\begin{bmatrix} \left(\tilde{\Psi}_0\right) \\ \left(\frac{\partial \tilde{\Psi}}{\partial r}\right) \\ \left(\frac{\partial \tilde{\Psi}}{\partial z}\right) \end{bmatrix}
= \begin{bmatrix} \Psi_A \\ \Psi_B \\ \Psi_C \end{bmatrix}
\tag{A16}
$$

These can be readily solved in closed form to find the appropriate coefficient parameter values. For the partial derivative coefficient parameters the solutions are as follows:

$$
\begin{aligned}
\left(\frac{\partial \tilde{I}}{\partial r}\right) &= \frac{I_A(z_B - z_C) + I_B(z_C - z_A) + I_C(z_A - z_B)}{r_A(z_B - z_C) + r_B(z_C - z_A) + r_C(z_A - z_B)} \\
\left(\frac{\partial \tilde{I}}{\partial z}\right) &= \frac{I_A(r_B - r_C) + I_B(r_C - r_A) + I_C(r_A - r_B)}{z_A(r_B - r_C) + z_B(r_C - r_A) + z_C(r_A - r_B)} \\
\left(\frac{\partial \tilde{\Psi}}{\partial r}\right) &= \frac{\Psi_A(z_B - z_C) + \Psi_B(z_C - z_A) + \Psi_C(z_A - z_B)}{r_A(z_B - z_C) + r_B(z_C - z_A) + r_C(z_A - z_B)} \\
\left(\frac{\partial \tilde{\Psi}}{\partial z}\right) &= \frac{\Psi_A(r_B - r_C) + \Psi_B(r_C - r_A) + \Psi_C(r_A - r_B)}{z_A(r_B - r_C) + z_B(r_C - r_A) + z_C(r_A - r_B)}
\end{aligned}
\tag{A17}
$$

Using this approximation the previous integrand becomes:

$$\hat{\theta} \bullet (\nabla I \times \nabla \Psi) \equiv \left(\frac{\partial I}{\partial r}\frac{\partial \Psi}{\partial z} - \frac{\partial I}{\partial z}\frac{\partial \Psi}{\partial r} \right) \approx \left(\frac{\partial \tilde{I}}{\partial r}\frac{\partial \tilde{\Psi}}{\partial z} - \frac{\partial \tilde{I}}{\partial z}\frac{\partial \tilde{\Psi}}{\partial r} \right) = \\ = \frac{I_A(\Psi_B - \Psi_C) + I_B(\Psi_C - \Psi_A) + I_C(\Psi_A - \Psi_B)}{r_A(z_B - z_C) + r_B(z_C - z_A) + r_C(z_A - z_B)} \tag{A18}$$

Note that as a result of assuming a linear model over the triangle, this approximation gives a constant value of the integrand over the triangle. Thus, the *integral* over the triangle is simply this integrand's constant value times the triangle's area, which, assuming the ABC point sequence is counterclockwise, is:

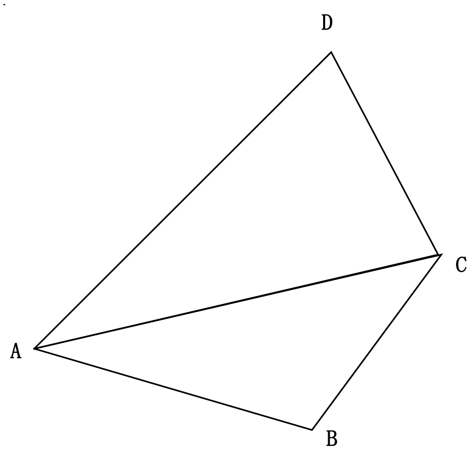
$$[\text{Area of Triangle } \Delta\text{ABC}] = \frac{1}{2}\begin{vmatrix} 1 & r_A & z_A \\ 1 & r_B & z_B \\ 1 & r_C & z_C \end{vmatrix} = \frac{r_A(z_B - z_C) + r_B(z_C - z_A) + r_C(z_A - z_B)}{2} \tag{A19}$$

This cancels all (r,z) coordinates, so the integral over the triangle becomes as stated in Eq.(A20).

$$\vec{M}_{\Delta ABC} \approx \hat{z}\left(\frac{I_A(\Psi_B - \Psi_C) + I_B(\Psi_C - \Psi_A) + I_C(\Psi_A - \Psi_B)}{4\pi} \right) \tag{A20}$$

This formula can be applied to an adjacent bordering triangle, ΔACD, by changing indices as in Eq.(A21).

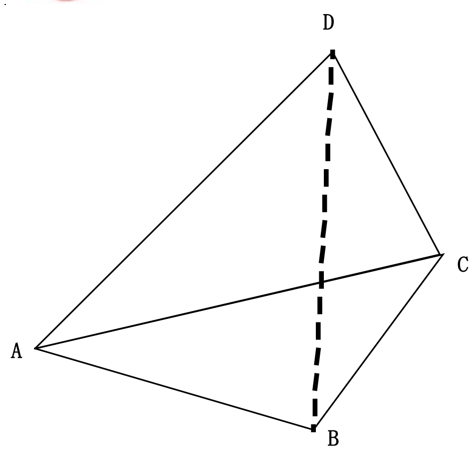
$$\vec{M}_{\Delta ACD} \approx \hat{z}\left(\frac{I_A(\Psi_C - \Psi_D) + I_C(\Psi_D - \Psi_A) + I_D(\Psi_A - \Psi_C)}{4\pi} \right) \tag{A21}$$



The two triangles together form a quadrilateral, ABCD, so the integral over the entire quadrilateral is the sum of the two triangle integrals:

$$\vec{M}_{ABCD} = \vec{M}_{\Delta ABC} + \vec{M}_{\Delta ACD} \approx \\ \approx \hat{z}\left(\begin{array}{c} \frac{I_A(\Psi_B - \Psi_C) + I_B(\Psi_C - \Psi_A) + I_C(\Psi_A - \Psi_B)}{4\pi} \\ + \frac{I_A(\Psi_C - \Psi_D) + I_C(\Psi_D - \Psi_A) + I_D(\Psi_A - \Psi_C)}{4\pi} \end{array} \right) = \\ = \hat{z}\left(\frac{(I_A - I_C)(\Psi_B - \Psi_D) - (I_B - I_D)(\Psi_A - \Psi_C)}{4\pi} \right) \tag{A22}$$

The quadrilateral, ABCD, could alternatively be decomposed in a different way into two triangles, i.e., into triangle ∆ABD and triangle ∆BCD. Although the linear coefficient parameter sets that would apply for these triangles would be different from the above, the final resulting formula for the moment integral over the quadrilateral ABCD turns out to be identically the same!



Note that in the important special case wherein the quadrilateral's corner points are located on two TF current stream function contour lines, it follows that $I_A=I_D$ and $I_B=I_C$. In that case the approximate formula for net torque in the quadrilateral region of the poloidal half-plane becomes simplified to Eq.(A23).

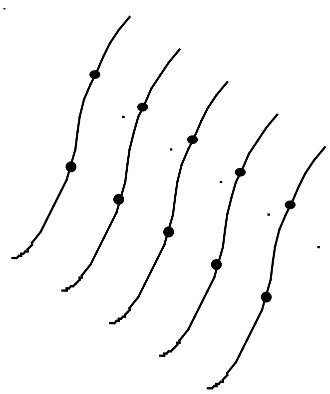
$$\vec{M}_{\text{ABCD}}^{\text{AD\&BC on TFCurrent Streamlines}} = \hat{z}\left(I_A - I_B\right)\left(\frac{\Psi_A + \Psi_B - \Psi_C - \Psi_D}{4\pi}\right). \tag{A23}$$

This is the simple average of the differences between per radian poloidal magnetic fluxes at the two ends of each of the two bounding current stream function contours, multiplied by the TF current enclosed between those two current stream function contours.

Thus, the torsional OOP loading of the TF coil system can be evaluated by taking simple sums and differences of poloidal flux evaluated at points located on TF current streamlines.

A Higher Accuracy Numerical Approximation

In the present case of this memo's calculations of net torque in the NSTX CSU TF conductor due to poloidal field interactions with TF current, five TF current streamlines have been chosen and their separation in terms of contour levels is the total TF current divided by four. The adjacent diagram illustrates the five contours and flux evaluation points on those contours for two adjacent poloidal angle locations. Applying the torque formula to each of the for quadrilaterals having the ten indicated locations as their corners, the sum of the net torque is as stated in Eq.(A24).



$$\begin{aligned}\vec{M} &= \hat{z}\left(\frac{I_{TF}}{4}\right)\left(\frac{\Psi_1^1 + \Psi_2^1 - \Psi_2^2 - \Psi_1^2}{4\pi}\right) + \hat{z}\left(\frac{I_{TF}}{4}\right)\left(\frac{\Psi_2^1 + \Psi_3^1 - \Psi_3^2 - \Psi_2^2}{4\pi}\right) \\ &\quad + \hat{z}\left(\frac{I_{TF}}{4}\right)\left(\frac{\Psi_3^1 + \Psi_4^1 - \Psi_4^2 - \Psi_3^2}{4\pi}\right) + \hat{z}\left(\frac{I_{TF}}{4}\right)\left(\frac{\Psi_4^1 + \Psi_5^1 - \Psi_5^2 - \Psi_4^2}{4\pi}\right) \end{aligned} \tag{A24}$$

$$= \hat{z}\frac{I_{TF}}{2\pi}\begin{pmatrix} 0.125\left(\Psi_1^1 - \Psi_1^2\right) + 0.25\left(\Psi_2^1 - \Psi_2^2\right) + 0.25\left(\Psi_3^1 - \Psi_3^2\right) \\ +0.25\left(\Psi_4^1 - \Psi_4^2\right) + 0.125\left(\Psi_5^1 - \Psi_5^2\right) \end{pmatrix}$$

Thus, in the sum over one poloidal interval we must premultiply the column vector of five local flux values by the row vector, [1 2 2 2 1], then divide by 16π before subtracting from the corresponding result for the other poloidal location.

Note that if all five flux values at one poloidal location were identical to each other the result would be that single value divided by 2π.

Note also that there is no benefit from repeating this summation procedure over many poloidal intervals in a TF conductor segment and summing their results, since the quantities evaluated at all intermediate poloidal locations would identically cancel each other out in taking the poloidal intervals sum. It is mathematically equivalent to directly subtract the quantities calculated on the five contours at the TF conductor segment's ends.

APPENDIX 2

Torsion Analysis Results With Nominal Stiffness Parameter Values

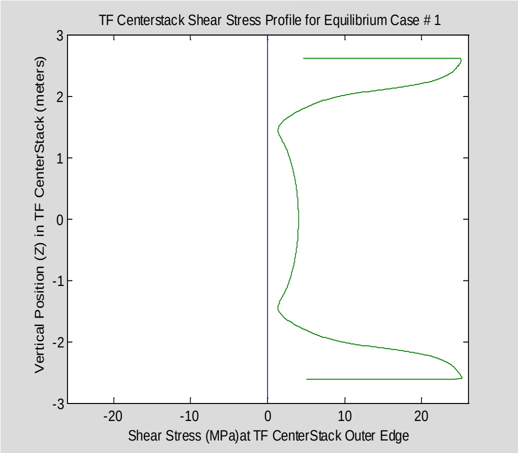
Using the stiffness parameter values of this memo, hear stress stress profiles were calculated for the 96 plasma equilibrium cases previously defined by J. Menard and for a single +24 kA OH precharge case. The different shear profiles are all graphed on the following pages. Peak shear stress values are collected in the following table for these cases. The maximum absolute shear was 25.234 MPa, observed for equilibrium case #1 and again in equilibrium case #16.

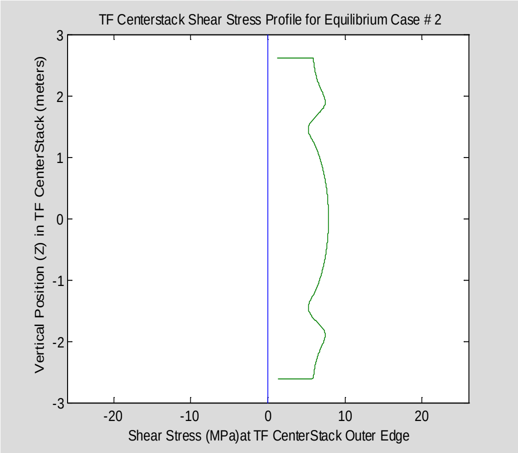
Table A2 and plots show peak shear stress is mostly due to OH, and reaches its maximum when the OH current is -24 kA. PF coils and plasma are smaller contributors to peak shear stress.

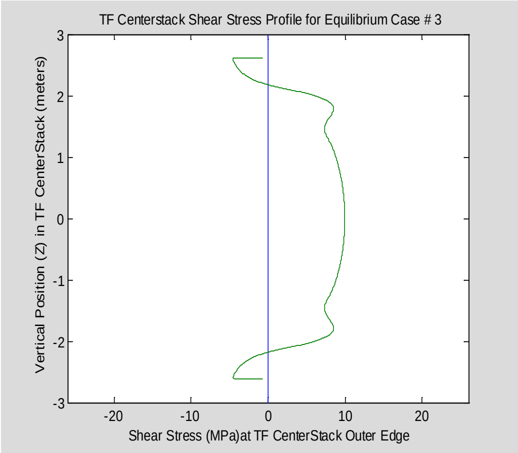
Table A2: TF Centerstack Max Absolute Shear Stress ; Precharge and Plasma Cases

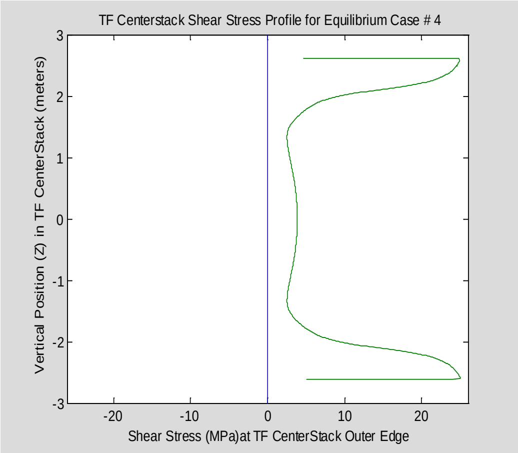
Case# -24 kA OH	Peak Shear MPa	Case# 0 kA OH	Peak Shear MPa	Case# 13 kA OH	Peak Shear MPa	Case 24 kA OH	Peak Shear MPa
						Precharge	17.075
1	25.2342	2	7.8607	3	9.9531		
4	25.0242	5	7.7104	6	9.7952		
7	24.8916	8	7.8971	9	9.7799		
10	24.7792	11	8.1233	12	10.1388		
13	24.7974	14	8.1955	15	10.2218		
16	25.2342	17	7.8607	18	9.9531		
19	23.5284	20	7.5095	21	9.5377		
22	24.0073	23	7.7574	24	9.7794		
25	24.3660	26	7.8945	27	9.8867		
28	24.6358	29	8.1082	30	9.9733		
31	24.7383	32	8.4486	33	10.1411		
34	22.7757	35	9.6055	36	11.8424		
37	23.1024	38	9.6302	39	11.8662		
40	23.5114	41	9.5922	42	11.8414		
43	23.9825	44	9.4749	45	11.7359		
46	24.3098	47	9.1979	38	11.5040		
49	21.5879	50	8.5384	51	10.7784		
52	21.9790	53	8.6015	54	10.8492		
55	22.5061	56	8.6266	57	10.8669		
58	23.0224	59	8.5566	60	10.8048		
61	23.5439	62	8.3331	63	10.6176		
64	25.1894	65	8.9480	66	11.0566		
67	25.0718	68	8.7566	69	10.8486		
70	24.6347	71	8.1963	72	10.2964		
73	24.1861	74	7.8472	75	9.9392		
76	23.7807	77	7.5209	78	9.6075		
79	23.2512	80	7.2031	81	9.2976		
82	24.6500	83	10.1626	84	12.1672		

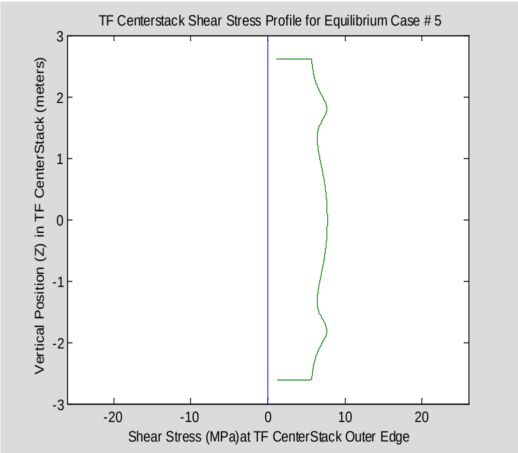
85	**24.7093**	86	**9.3389**	87	**11.3258**		
88	**24.7711**	89	**8.7861**	90	**10.7808**		
91	**24.8086**	92	**8.4007**	93	**10.3819**		
94	**24.8460**	95	**8.0646**	96	**10.0348**		

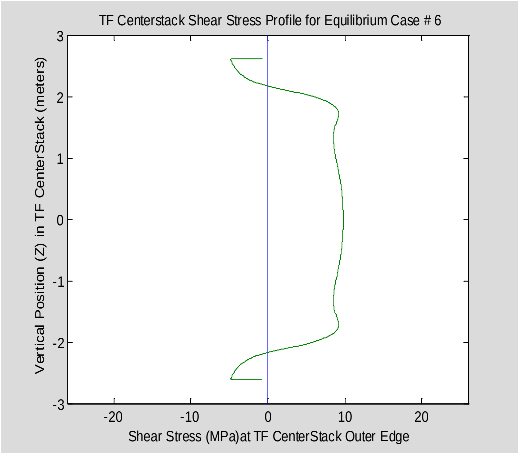

TF Centerstack Shear Stress Profile for Equilibrium Case # 1
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

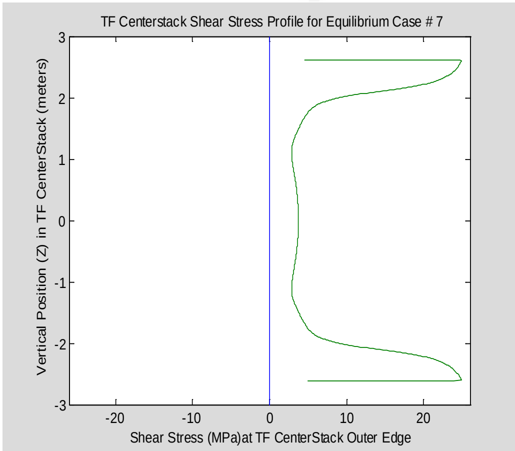

TF Centerstack Shear Stress Profile for Equilibrium Case # 2
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

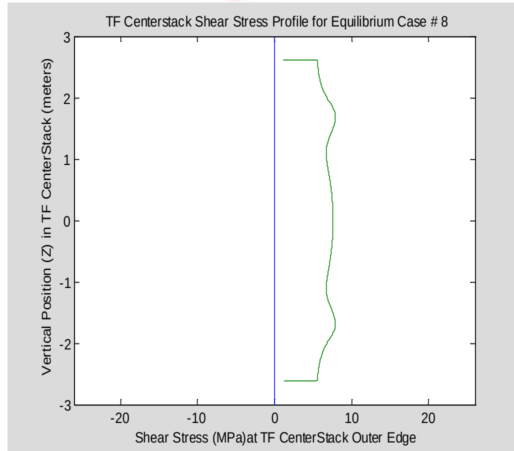

TF Centerstack Shear Stress Profile for Equilibrium Case # 3
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

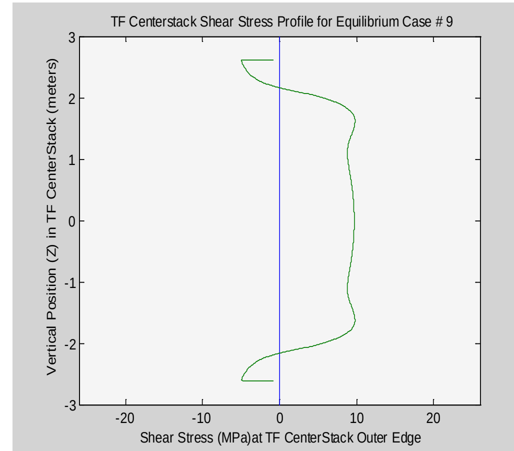

TF Centerstack Shear Stress Profile for Equilibrium Case # 4
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

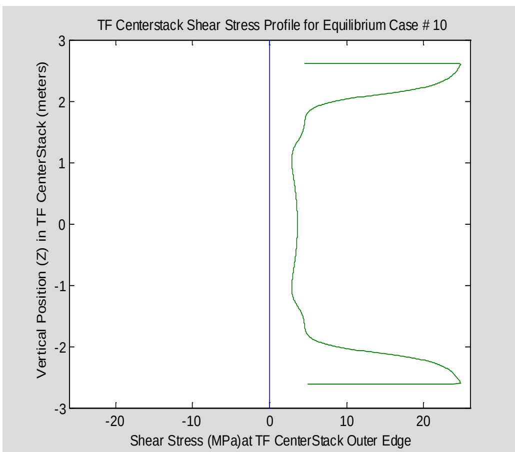

TF Centerstack Shear Stress Profile for Equilibrium Case # 5
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

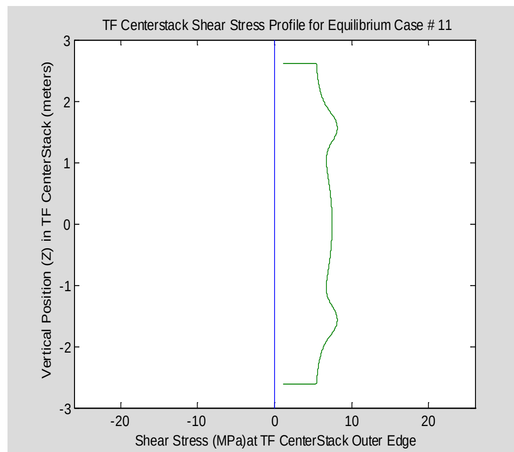

TF Centerstack Shear Stress Profile for Equilibrium Case # 6
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

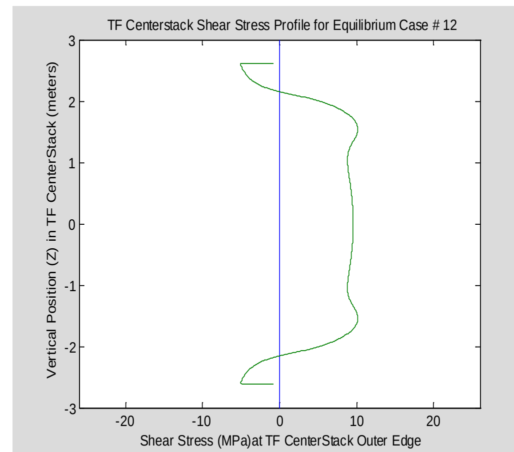

TF Centerstack Shear Stress Profile for Equilibrium Case # 7
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

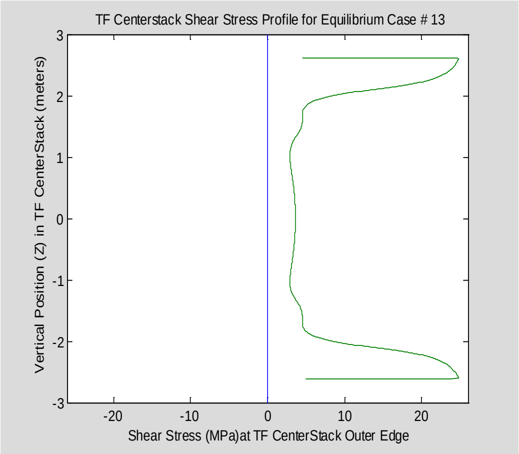
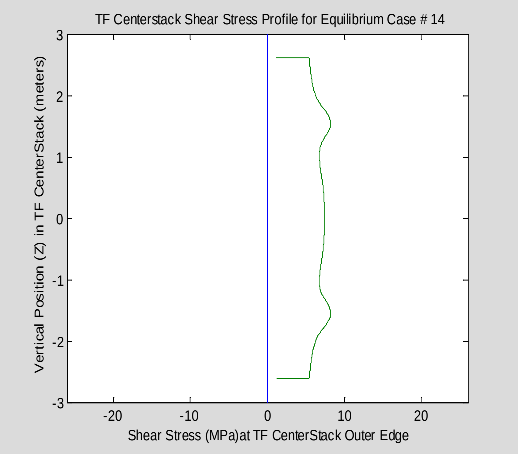
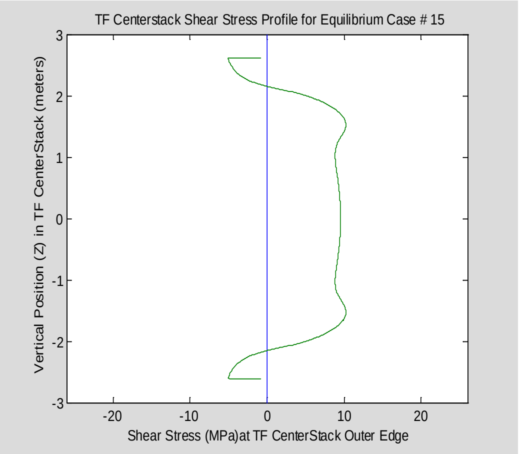
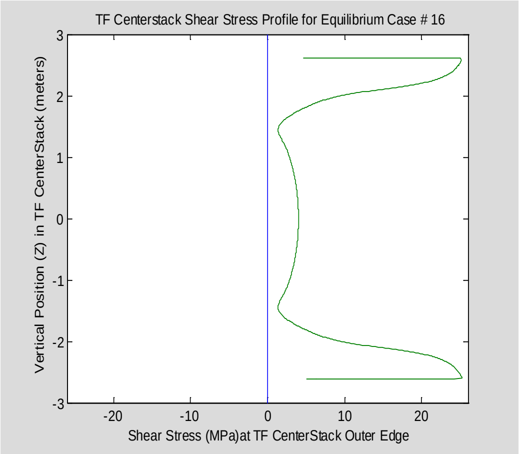
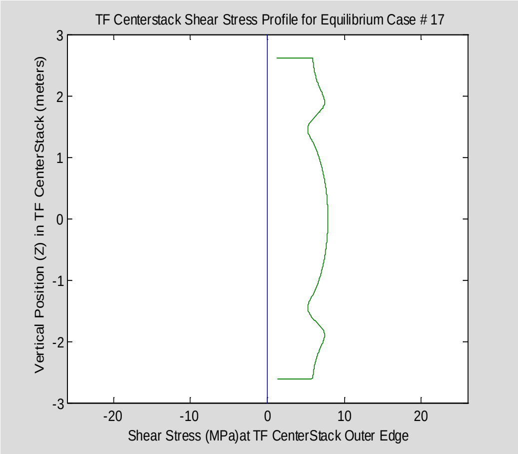
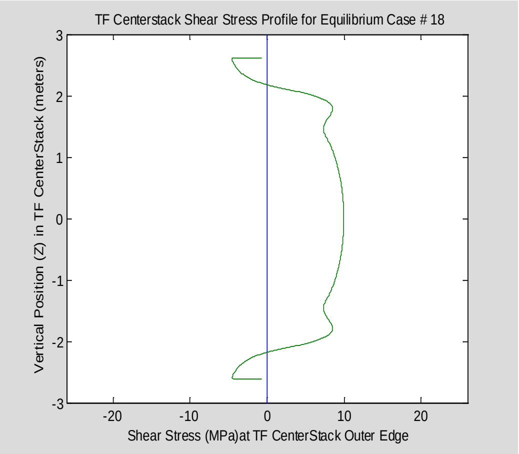

TF Centerstack Shear Stress Profile for Equilibrium Case # 8
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

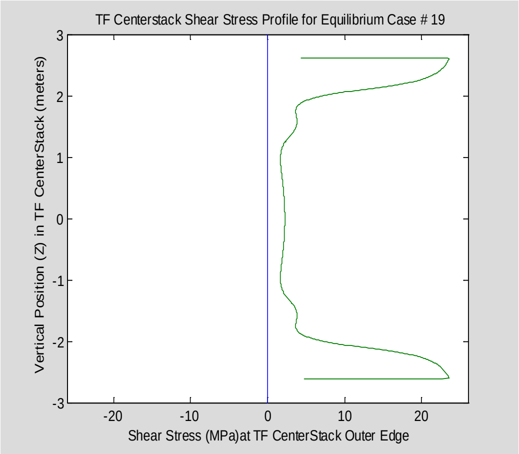

TF Centerstack Shear Stress Profile for Equilibrium Case # 9
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

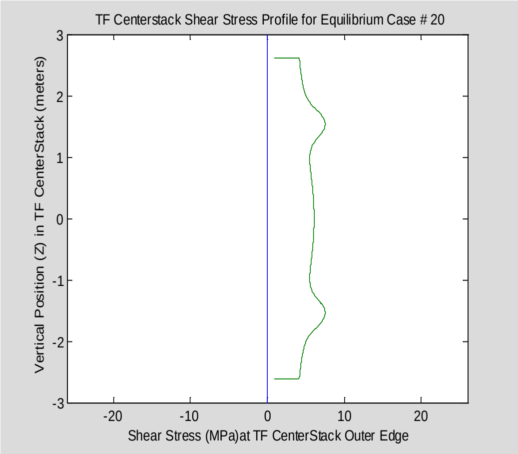

TF Centerstack Shear Stress Profile for Equilibrium Case # 10
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

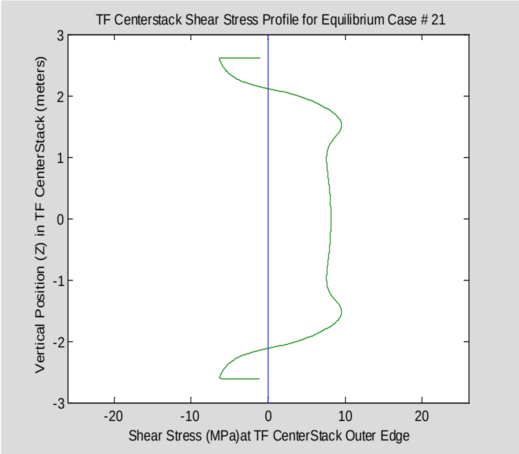

TF Centerstack Shear Stress Profile for Equilibrium Case # 11
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

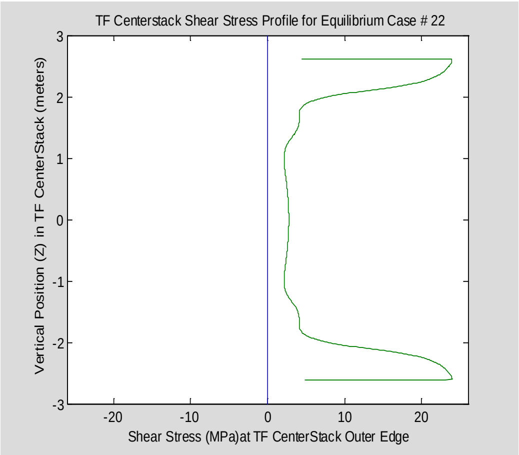

TF Centerstack Shear Stress Profile for Equilibrium Case # 12
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

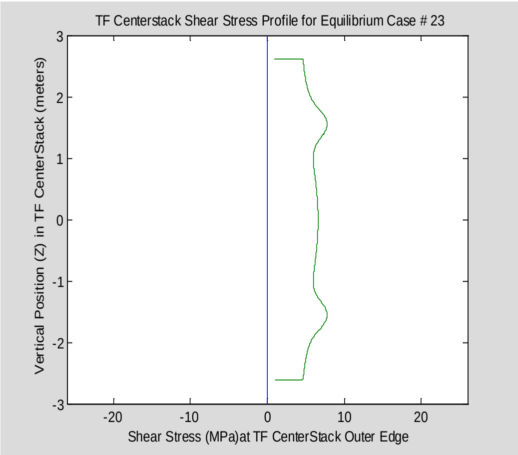

TF Centerstack Shear Stress Profile for Equilibrium Case # 13
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

TF Centerstack Shear Stress Profile for Equilibrium Case # 14
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

TF Centerstack Shear Stress Profile for Equilibrium Case # 15
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

TF Centerstack Shear Stress Profile for Equilibrium Case # 16
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

TF Centerstack Shear Stress Profile for Equilibrium Case # 17
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

TF Centerstack Shear Stress Profile for Equilibrium Case # 18
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

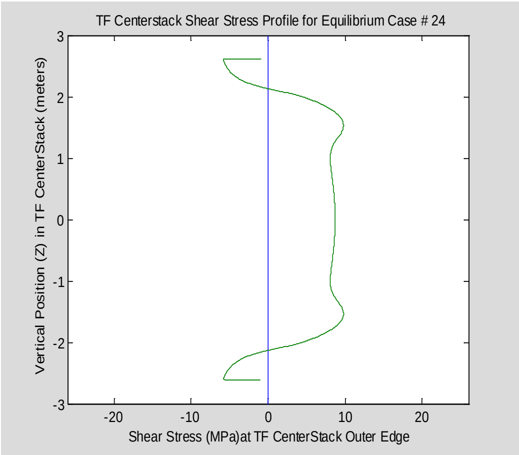

TF Centerstack Shear Stress Profile for Equilibrium Case # 19
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

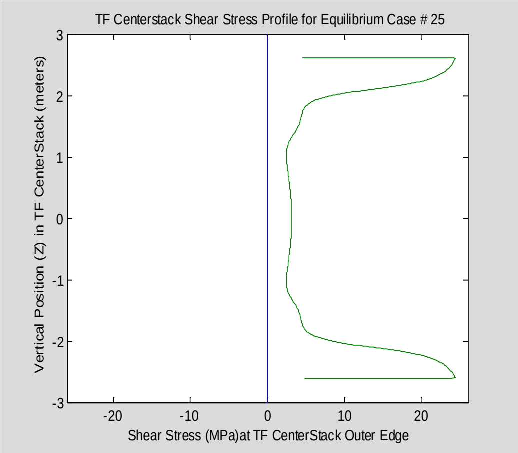

TF Centerstack Shear Stress Profile for Equilibrium Case # 20
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

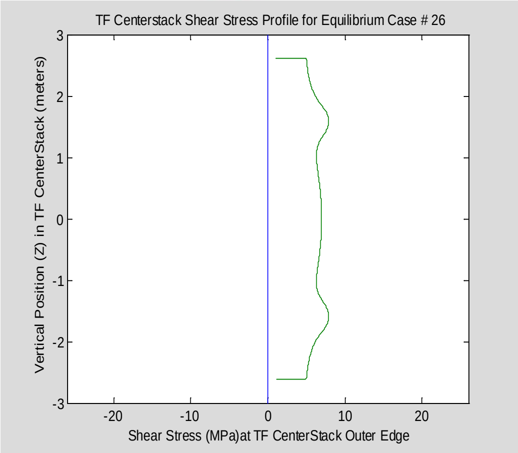

TF Centerstack Shear Stress Profile for Equilibrium Case # 21
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

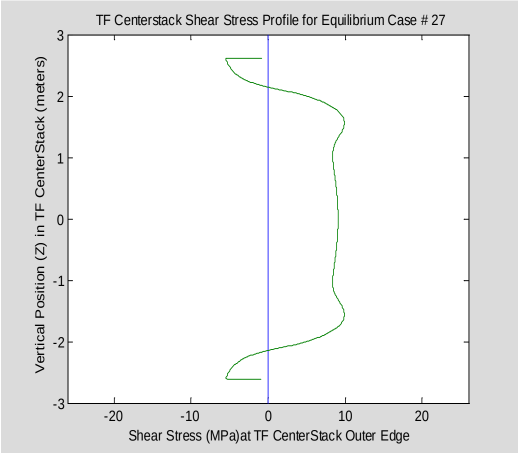

TF Centerstack Shear Stress Profile for Equilibrium Case # 22
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

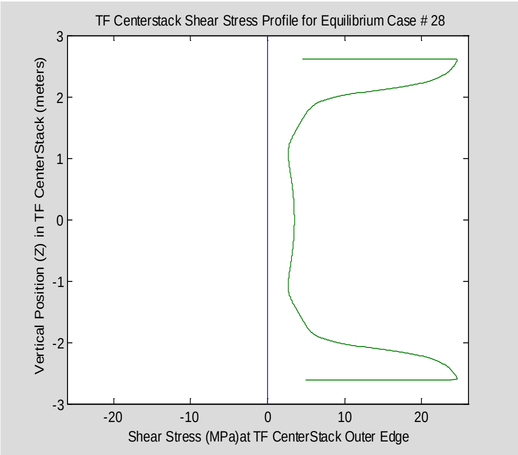

TF Centerstack Shear Stress Profile for Equilibrium Case # 23
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

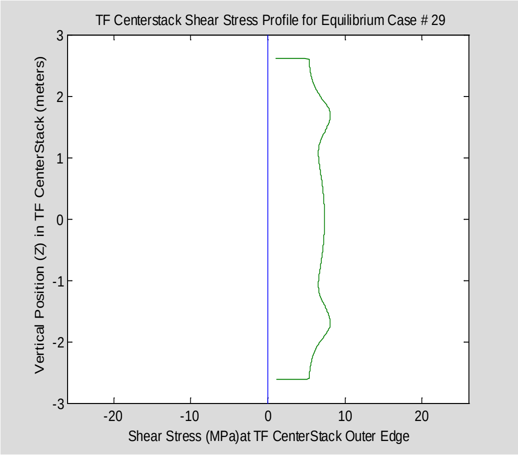

TF Centerstack Shear Stress Profile for Equilibrium Case # 24
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

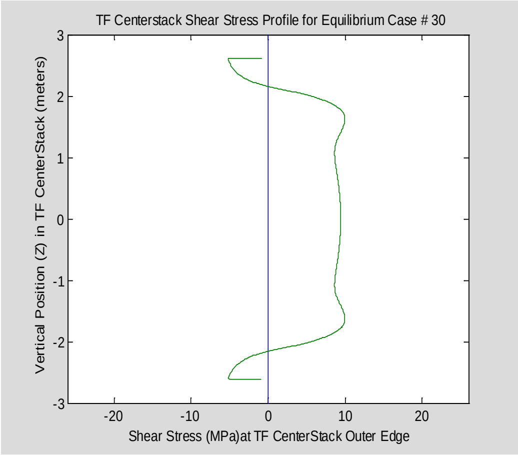

TF Centerstack Shear Stress Profile for Equilibrium Case # 25
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 26
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

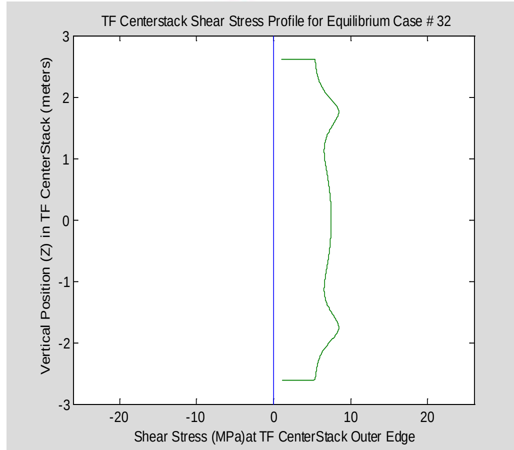

TF Centerstack Shear Stress Profile for Equilibrium Case # 27
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

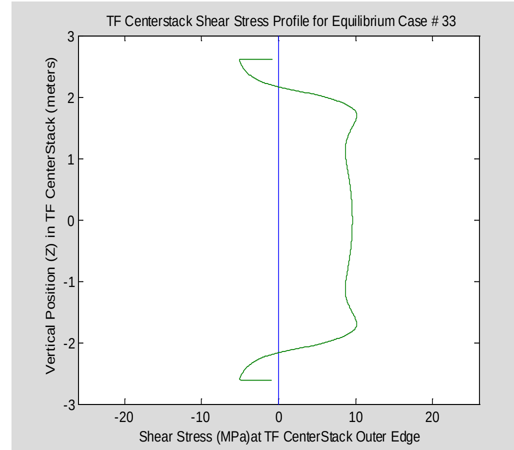

TF Centerstack Shear Stress Profile for Equilibrium Case # 28
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

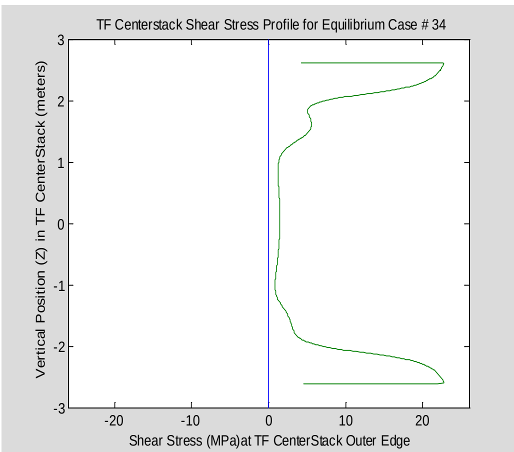

TF Centerstack Shear Stress Profile for Equilibrium Case # 29
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

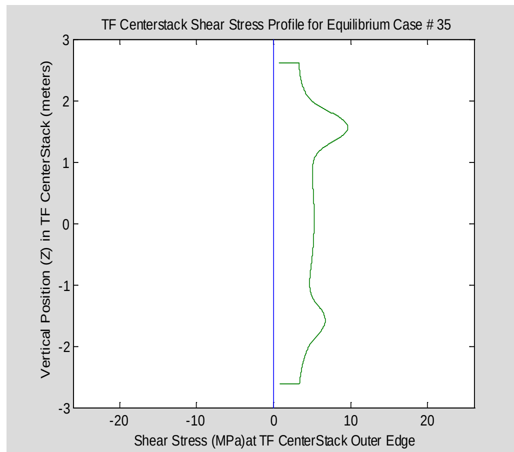

TF Centerstack Shear Stress Profile for Equilibrium Case # 30
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

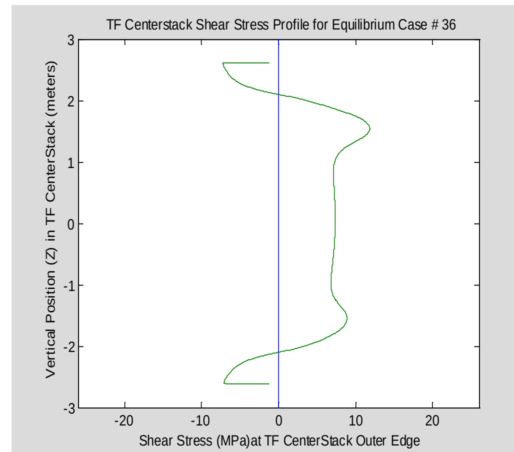

TF Centerstack Shear Stress Profile for Equilibrium Case # 31
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

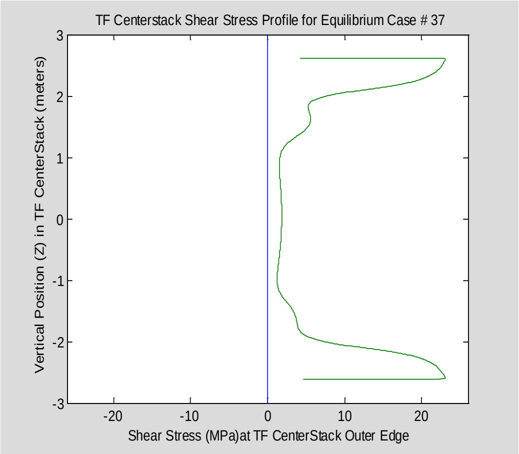

TF Centerstack Shear Stress Profile for Equilibrium Case # 32
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

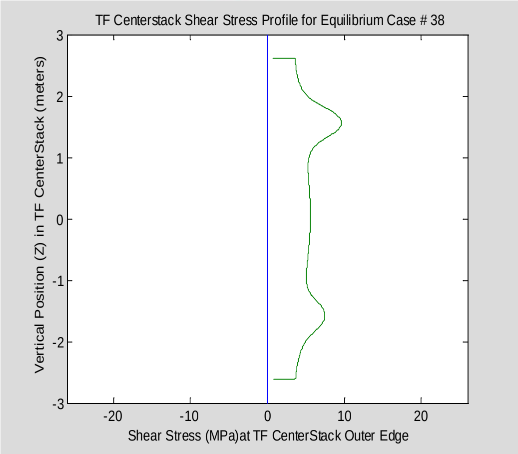

TF Centerstack Shear Stress Profile for Equilibrium Case # 33
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

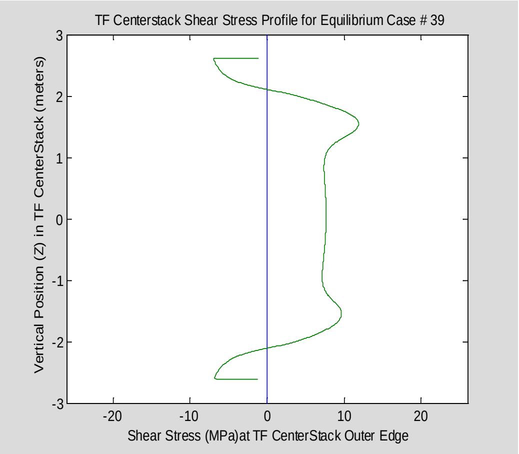

TF Centerstack Shear Stress Profile for Equilibrium Case # 34
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

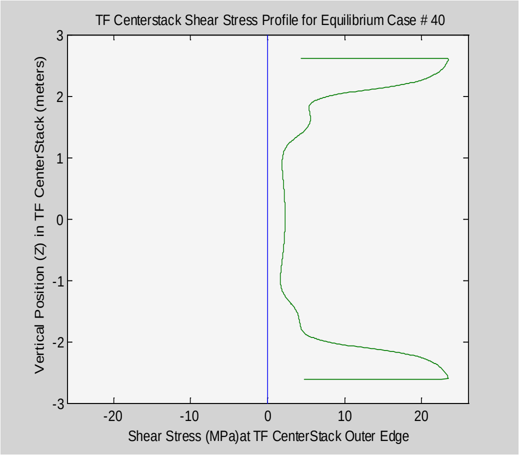

TF Centerstack Shear Stress Profile for Equilibrium Case # 35
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

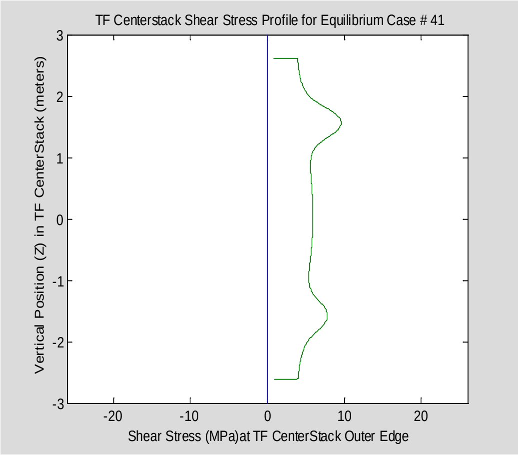

TF Centerstack Shear Stress Profile for Equilibrium Case # 36
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

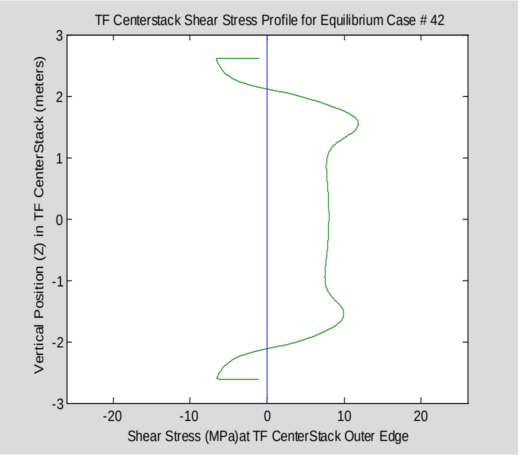

TF Centerstack Shear Stress Profile for Equilibrium Case # 37
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

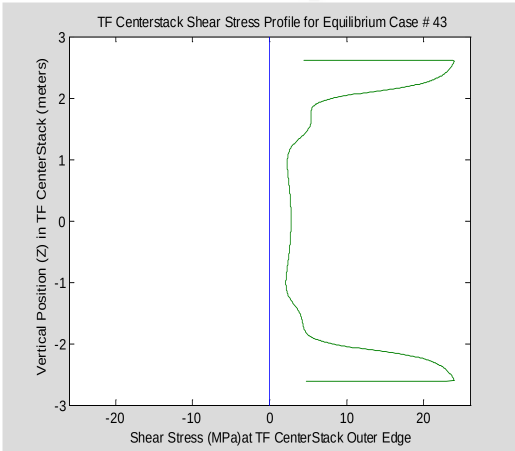

TF Centerstack Shear Stress Profile for Equilibrium Case # 38
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

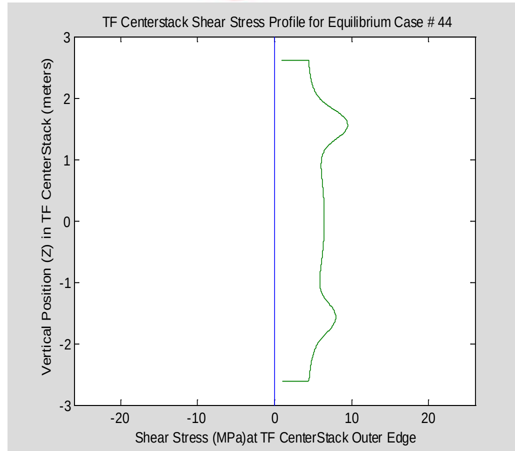

TF Centerstack Shear Stress Profile for Equilibrium Case # 39
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

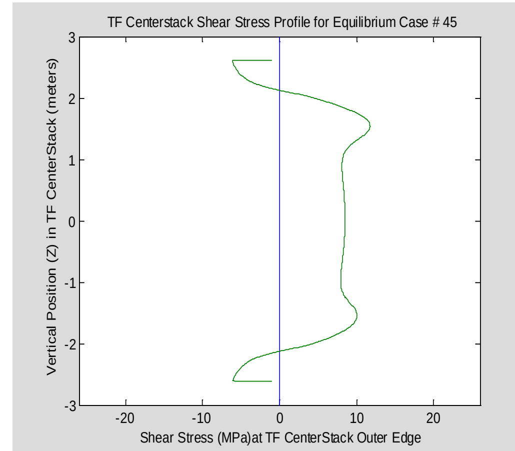

TF Centerstack Shear Stress Profile for Equilibrium Case # 40
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

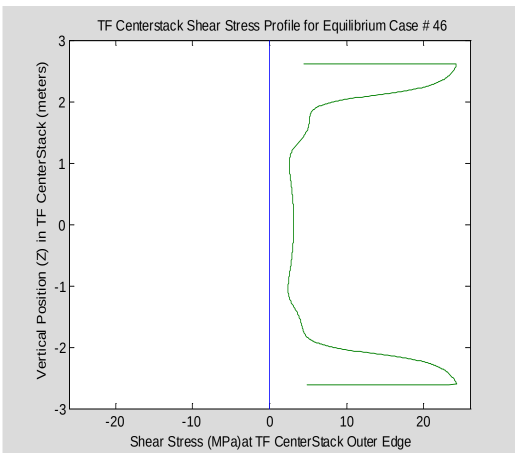

TF Centerstack Shear Stress Profile for Equilibrium Case # 41
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

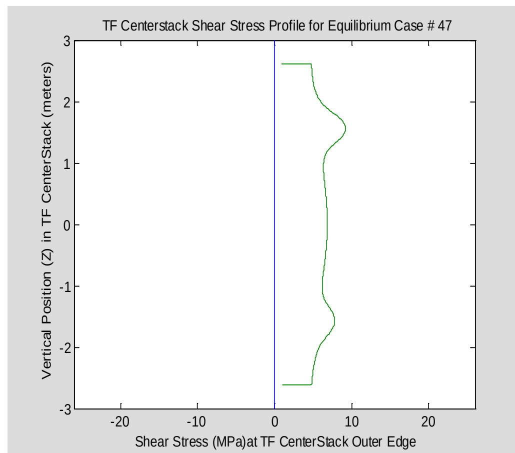

TF Centerstack Shear Stress Profile for Equilibrium Case # 42
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

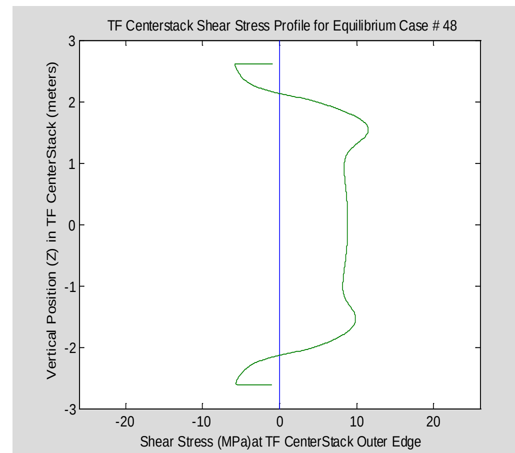

TF Centerstack Shear Stress Profile for Equilibrium Case # 43
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

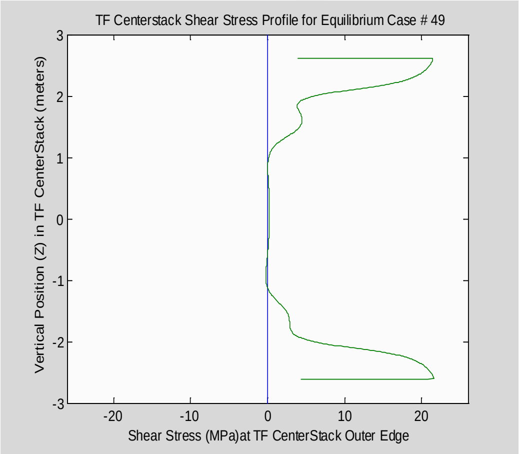

TF Centerstack Shear Stress Profile for Equilibrium Case # 44
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

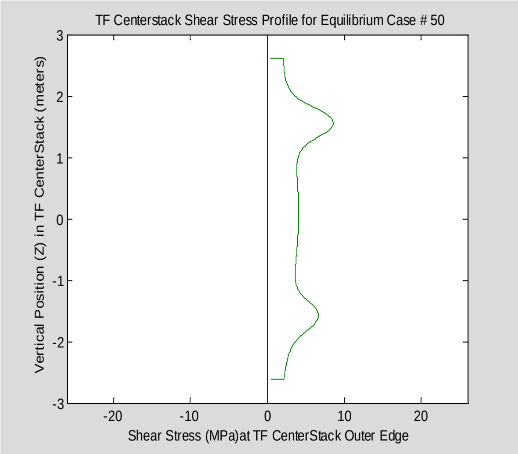

TF Centerstack Shear Stress Profile for Equilibrium Case # 45
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

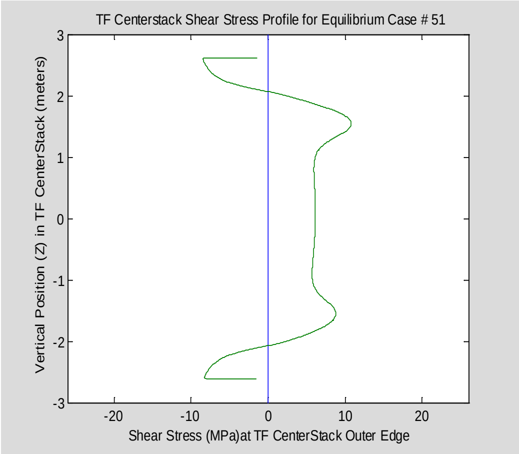

TF Centerstack Shear Stress Profile for Equilibrium Case # 46
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

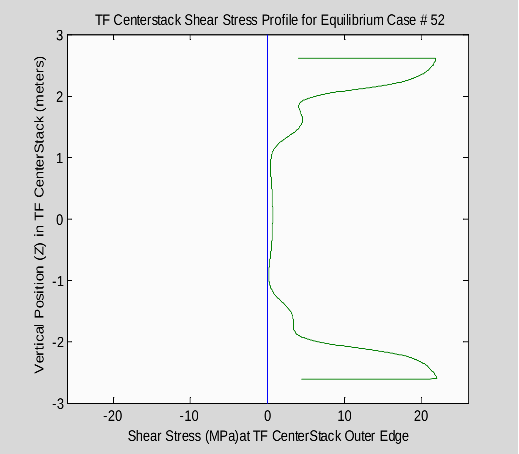

TF Centerstack Shear Stress Profile for Equilibrium Case # 47
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

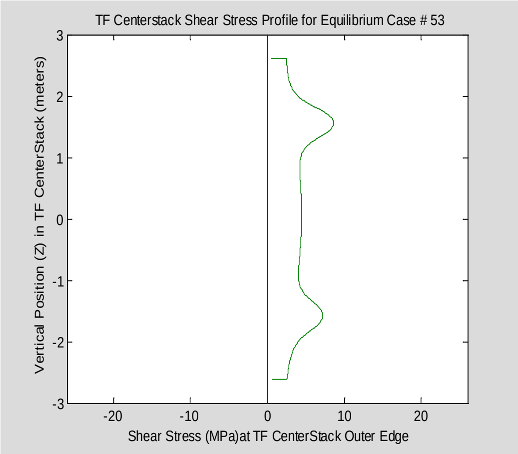

TF Centerstack Shear Stress Profile for Equilibrium Case # 48
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

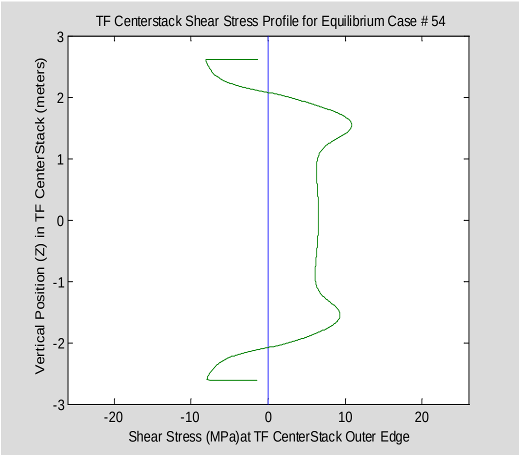

TF Centerstack Shear Stress Profile for Equilibrium Case # 49
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

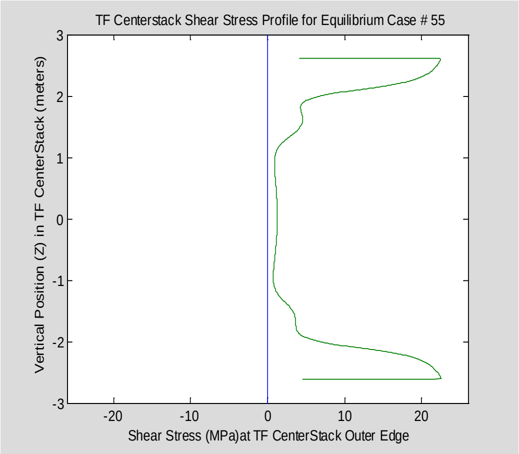

TF Centerstack Shear Stress Profile for Equilibrium Case # 50
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

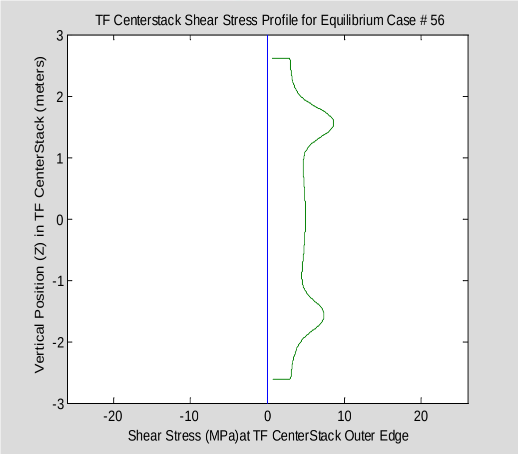

TF Centerstack Shear Stress Profile for Equilibrium Case # 51
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

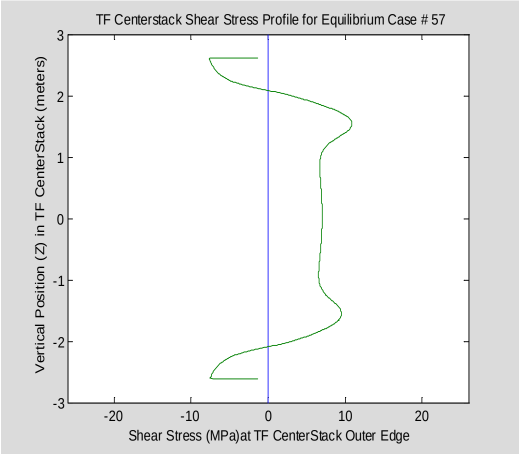

TF Centerstack Shear Stress Profile for Equilibrium Case # 52
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

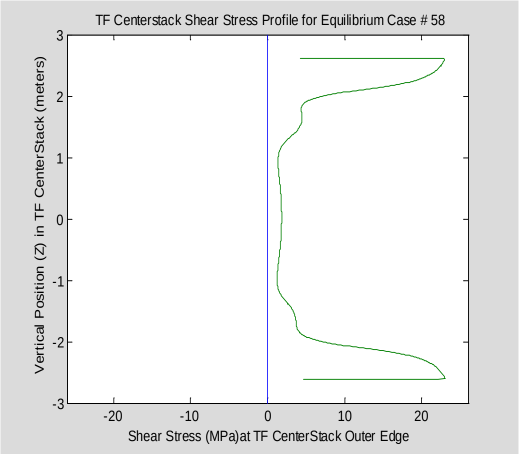

TF Centerstack Shear Stress Profile for Equilibrium Case # 53
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

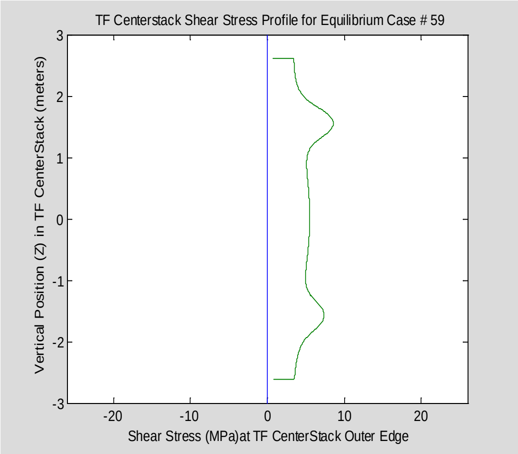

TF Centerstack Shear Stress Profile for Equilibrium Case # 54
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

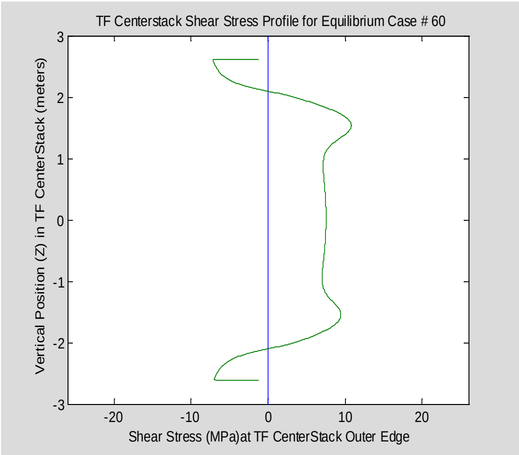

TF Centerstack Shear Stress Profile for Equilibrium Case # 55
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

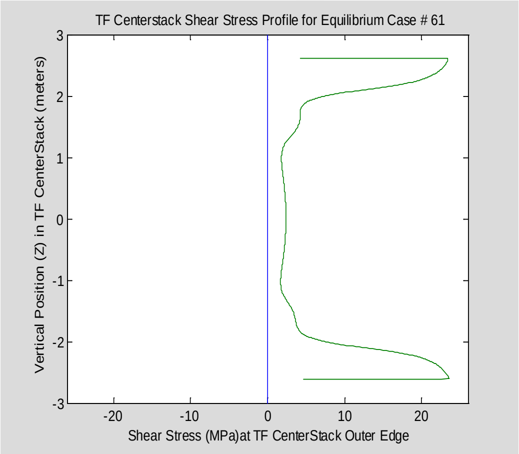

TF Centerstack Shear Stress Profile for Equilibrium Case # 56
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

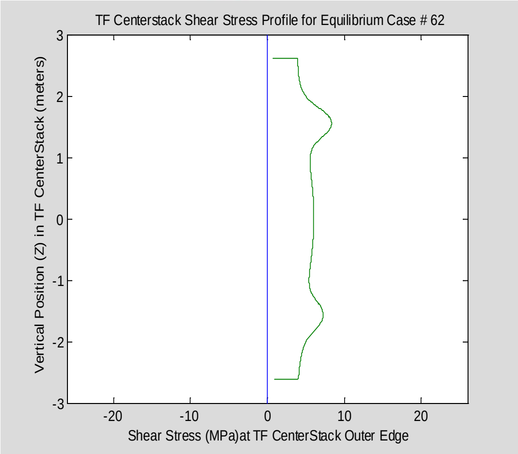

TF Centerstack Shear Stress Profile for Equilibrium Case # 57
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

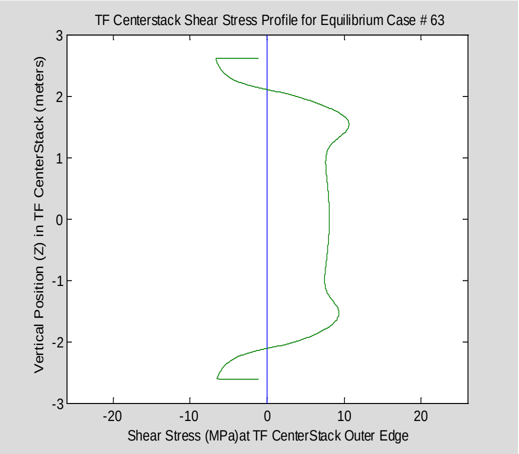

TF Centerstack Shear Stress Profile for Equilibrium Case # 58
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

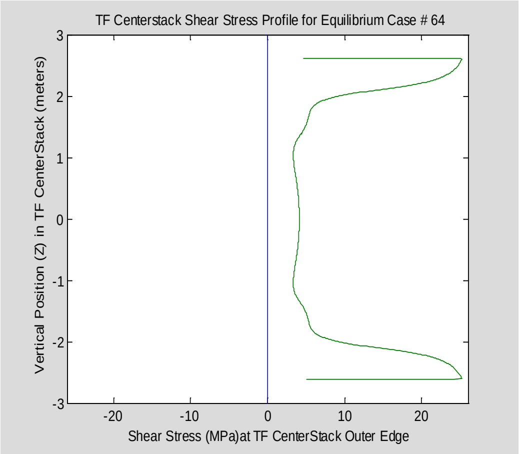

TF Centerstack Shear Stress Profile for Equilibrium Case # 59
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

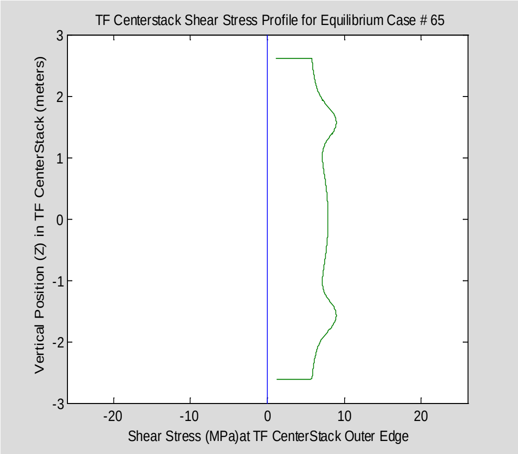

TF Centerstack Shear Stress Profile for Equilibrium Case # 60
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

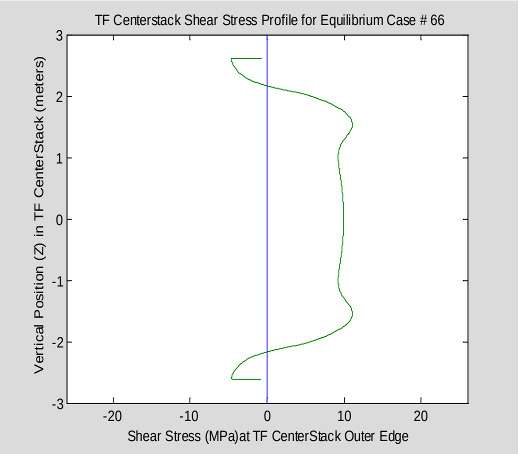

TF Centerstack Shear Stress Profile for Equilibrium Case # 61
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 62
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

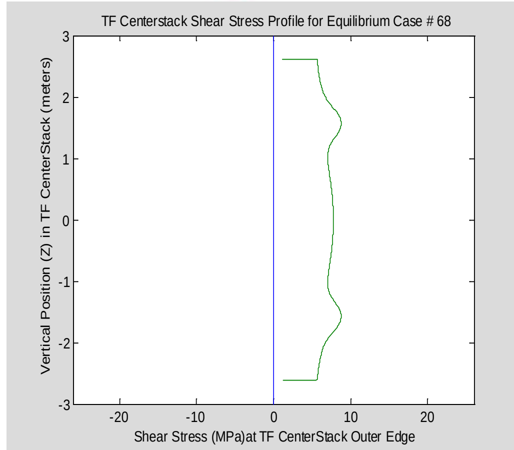

TF Centerstack Shear Stress Profile for Equilibrium Case # 63
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

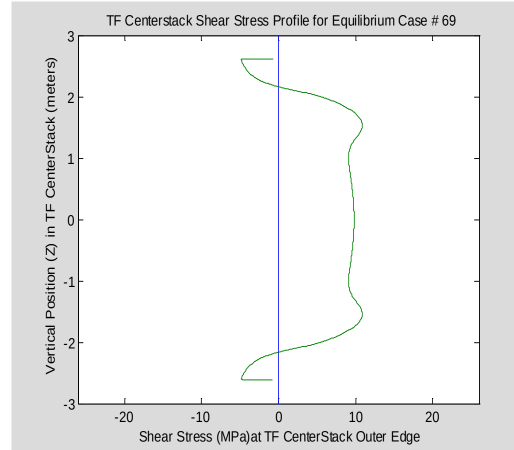

TF Centerstack Shear Stress Profile for Equilibrium Case # 64
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

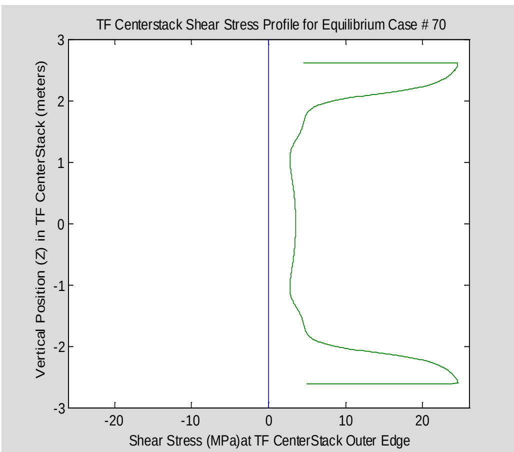

TF Centerstack Shear Stress Profile for Equilibrium Case # 65
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

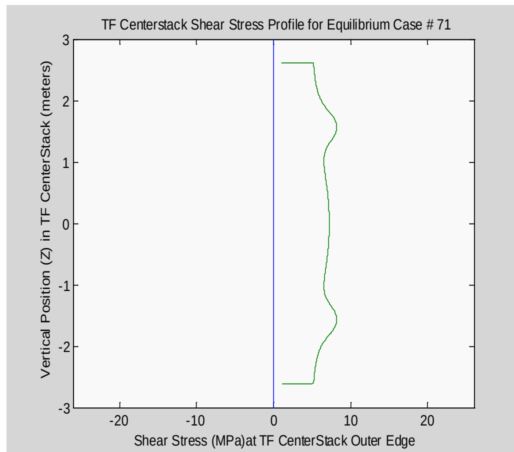

TF Centerstack Shear Stress Profile for Equilibrium Case # 66
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

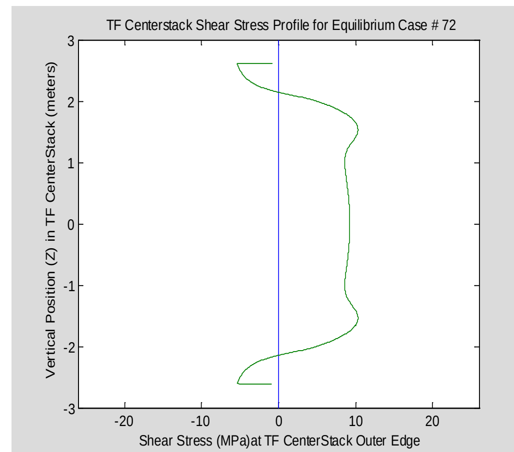

TF Centerstack Shear Stress Profile for Equilibrium Case # 67
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

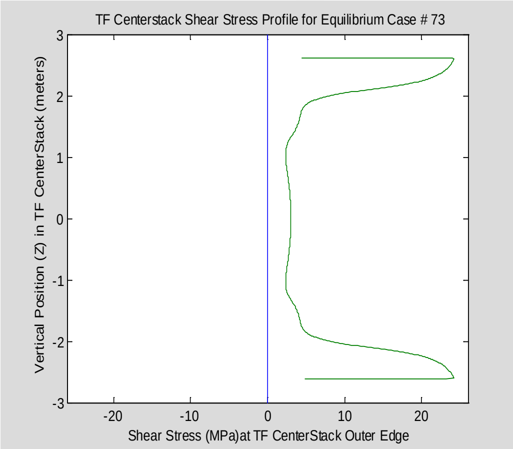

TF Centerstack Shear Stress Profile for Equilibrium Case # 68
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

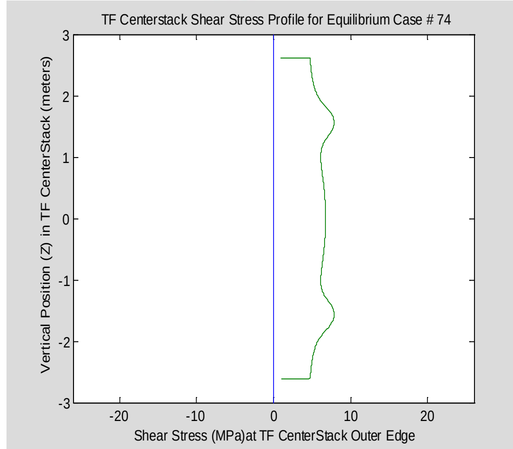

TF Centerstack Shear Stress Profile for Equilibrium Case # 69
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

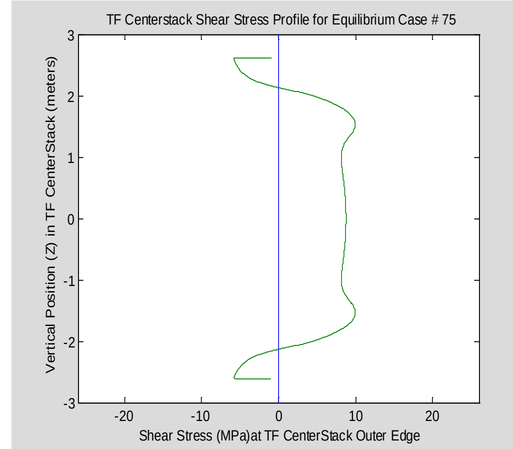

TF Centerstack Shear Stress Profile for Equilibrium Case # 70
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

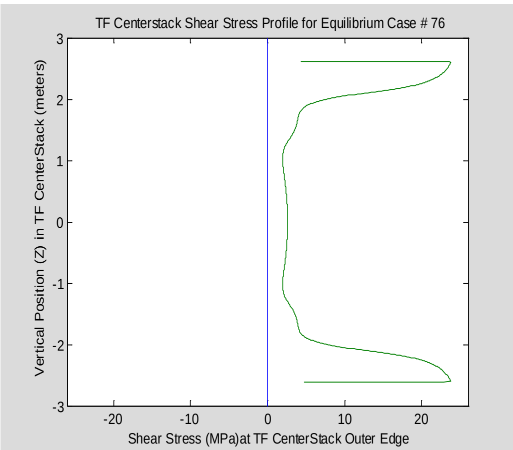

TF Centerstack Shear Stress Profile for Equilibrium Case # 71
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

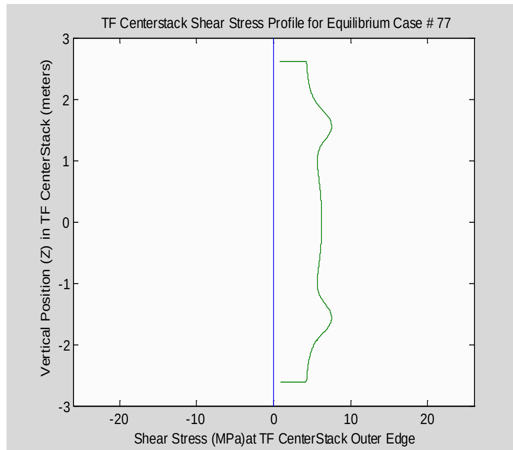

TF Centerstack Shear Stress Profile for Equilibrium Case # 72
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

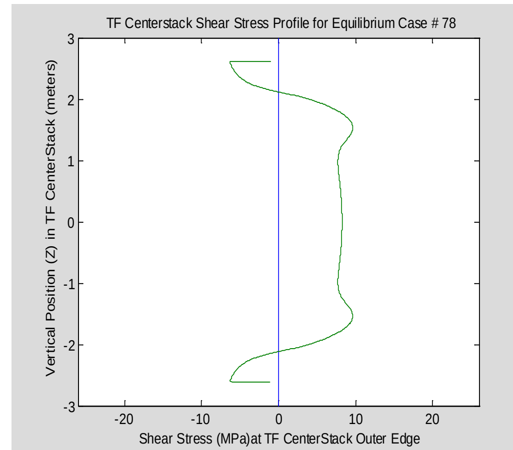

TF Centerstack Shear Stress Profile for Equilibrium Case # 73
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

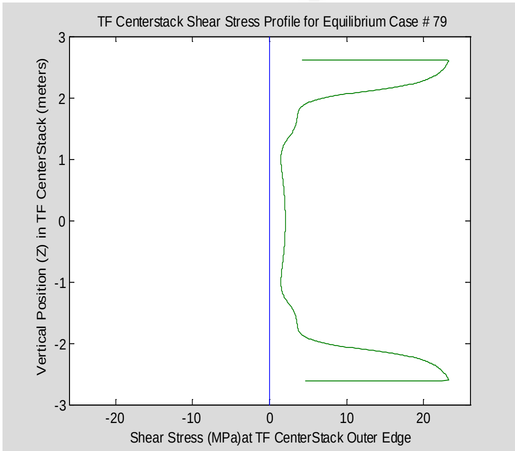

TF Centerstack Shear Stress Profile for Equilibrium Case # 74
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

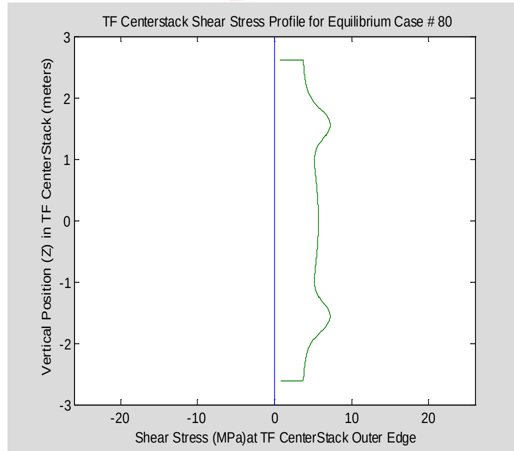

TF Centerstack Shear Stress Profile for Equilibrium Case # 75
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

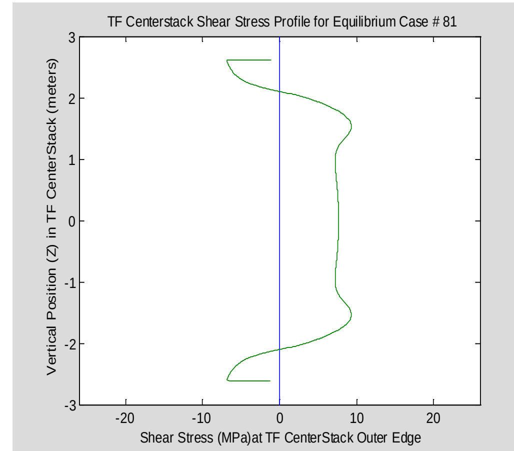

TF Centerstack Shear Stress Profile for Equilibrium Case # 76
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

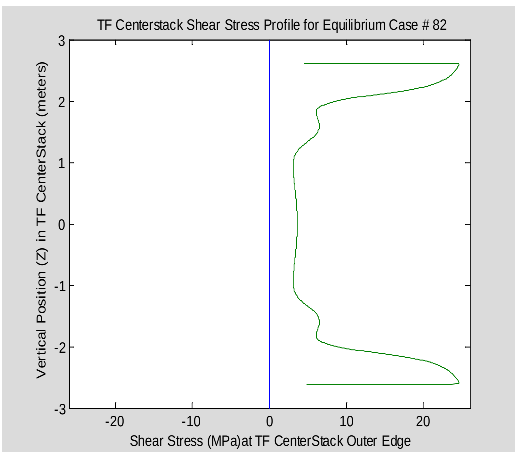

TF Centerstack Shear Stress Profile for Equilibrium Case # 77
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

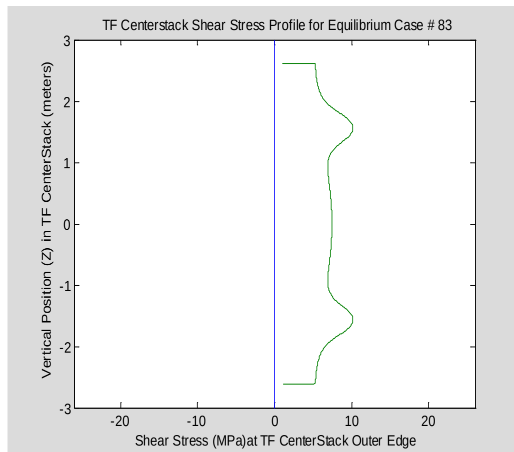

TF Centerstack Shear Stress Profile for Equilibrium Case # 78
Vertical Position (Z) in TF CenterStack (meters)
3
2
1
0
-1
-2
-3
-20
-10
0
10
20
Shear Stress (MPa)at TF CenterStack Outer Edge

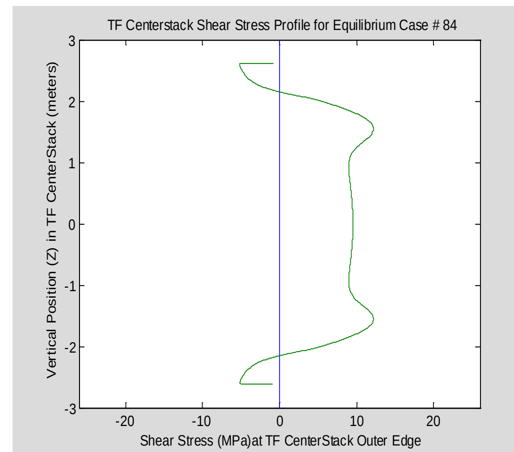

TF Centerstack Shear Stress Profile for Equilibrium Case # 79
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

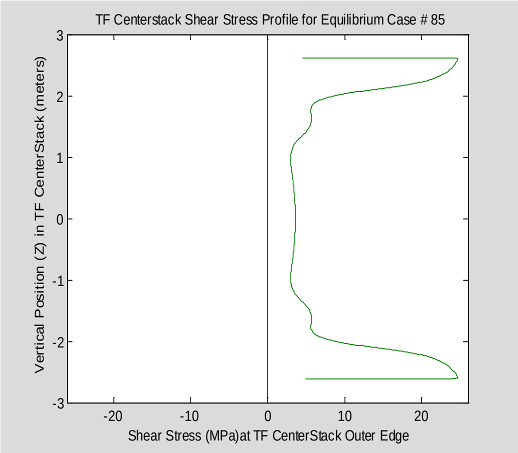

TF Centerstack Shear Stress Profile for Equilibrium Case # 80
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

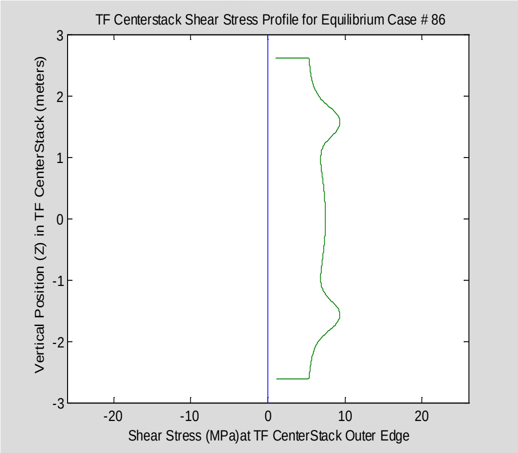

TF Centerstack Shear Stress Profile for Equilibrium Case # 81
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

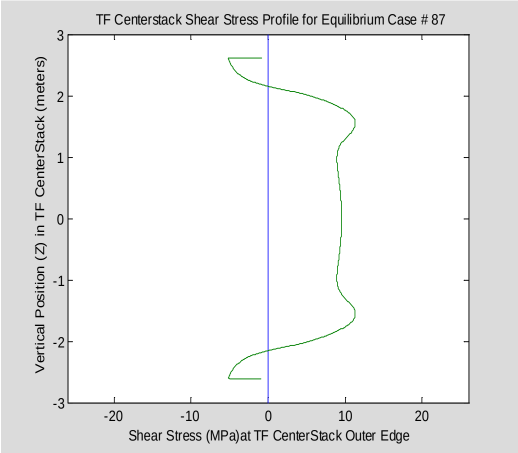

TF Centerstack Shear Stress Profile for Equilibrium Case # 82
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

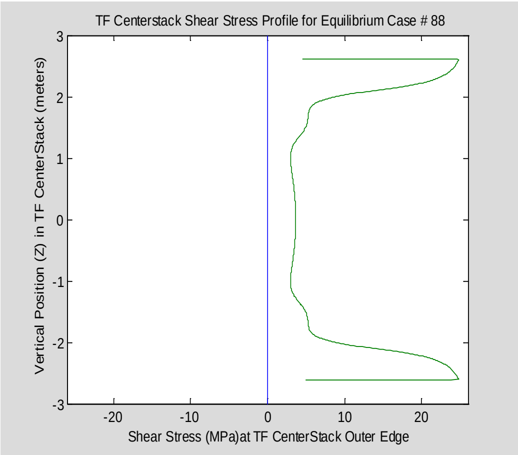

TF Centerstack Shear Stress Profile for Equilibrium Case # 83
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

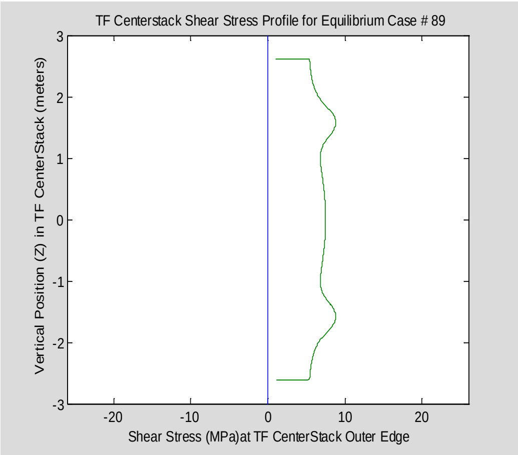

TF Centerstack Shear Stress Profile for Equilibrium Case # 84
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

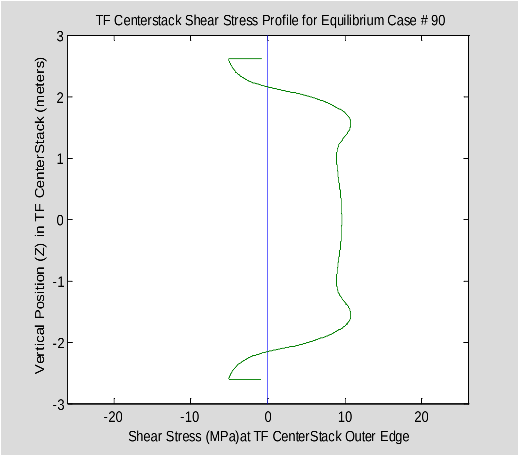

TF Centerstack Shear Stress Profile for Equilibrium Case # 85
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

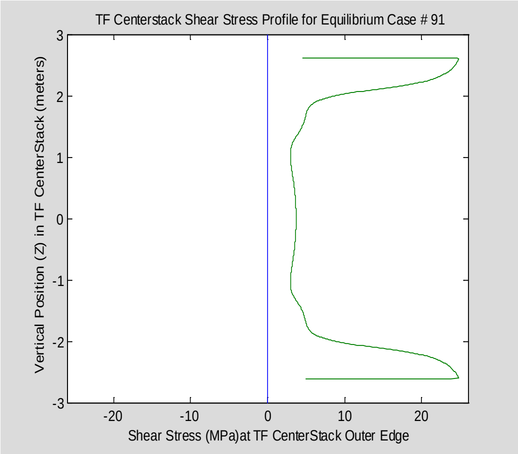

TF Centerstack Shear Stress Profile for Equilibrium Case # 86
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

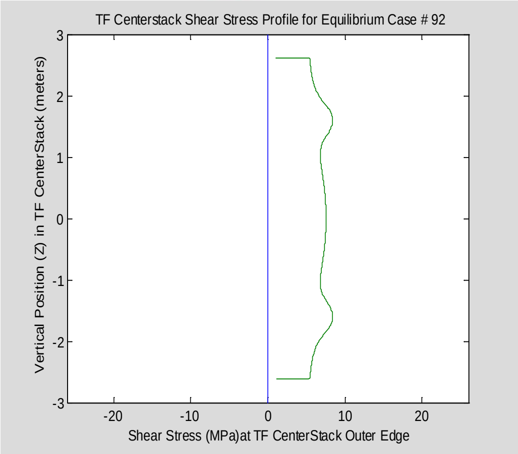

TF Centerstack Shear Stress Profile for Equilibrium Case # 87
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

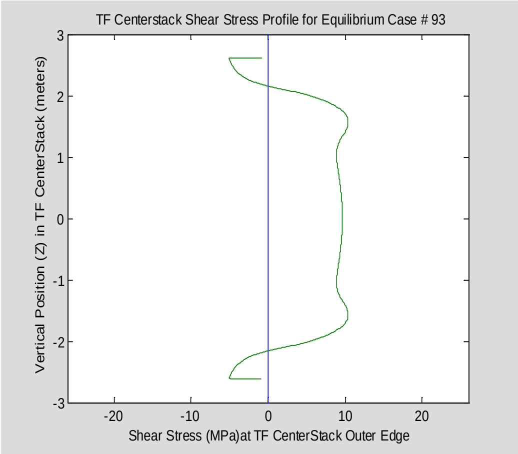

TF Centerstack Shear Stress Profile for Equilibrium Case # 88
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

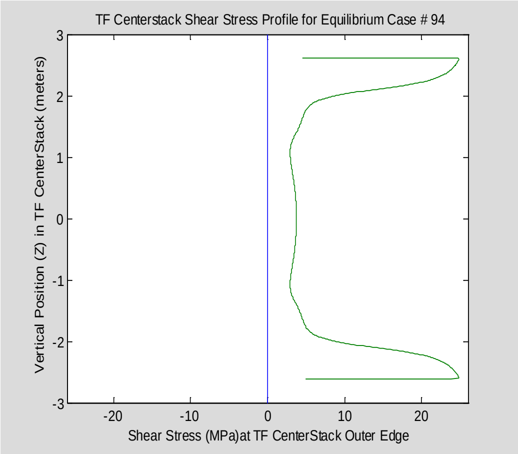

TF Centerstack Shear Stress Profile for Equilibrium Case # 89
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

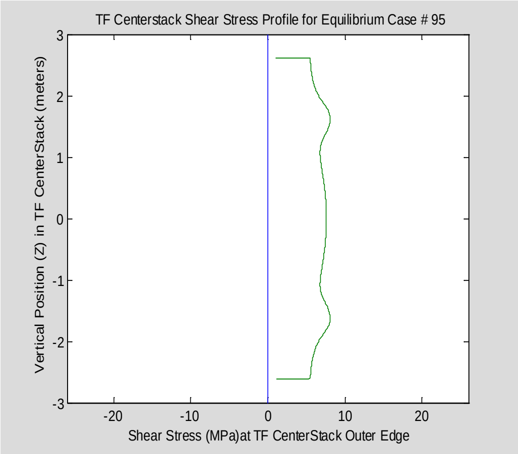

TF Centerstack Shear Stress Profile for Equilibrium Case # 90
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

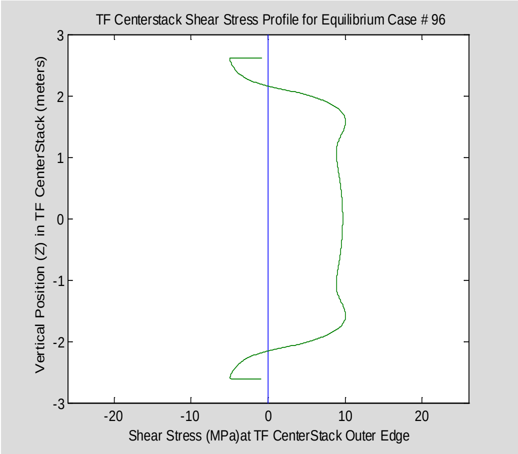

TF Centerstack Shear Stress Profile for Equilibrium Case # 91
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

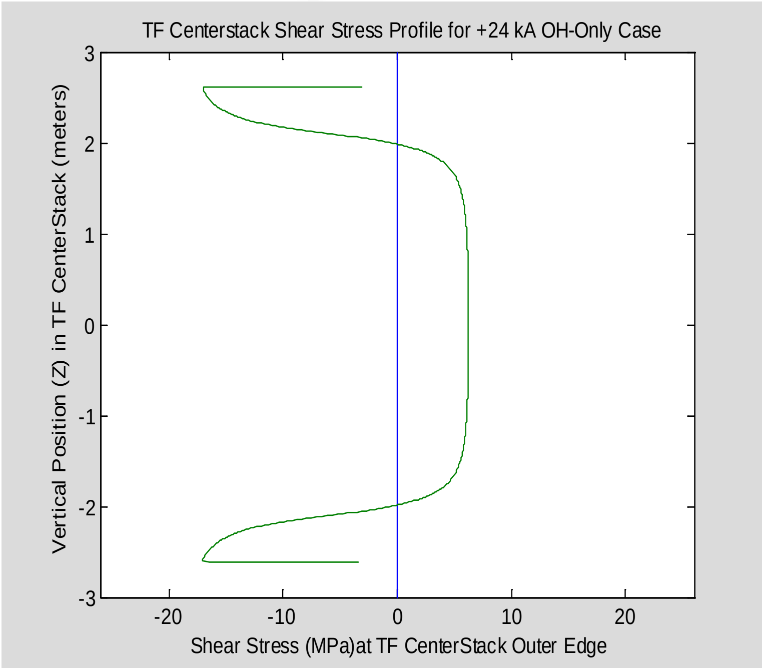

TF Centerstack Shear Stress Profile for Equilibrium Case # 92
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 93
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 94
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 95
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 96
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge



TF Centerstack Shear Stress Profile for +24 kA OH-Only Case
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-20
-10
0
10
20

APPENDIX 3

Torsion Analysis Results With Reduced Stiffness in Upper Lid

To determine the effects on peak shear stress, the torsional stiffness of the upper lid was reduced by a factor of 20 while all other stiffness parameters were left unchanged from the values used to calculate the Appendix 2 results.

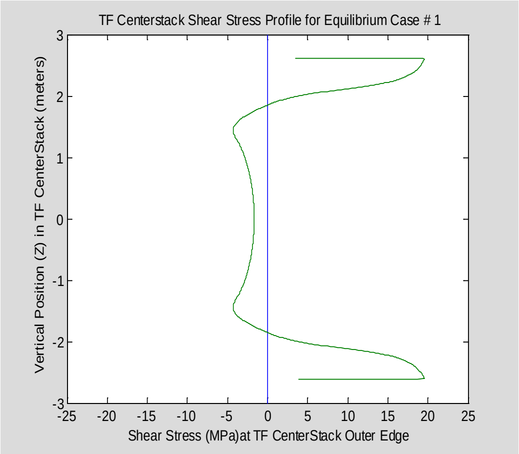
Peak shear stress values are collected in the following table for these cases. The maximum absolute shear was 19.553 MPa, observed for equilibrium case #1 and again in equilibrium case #16.

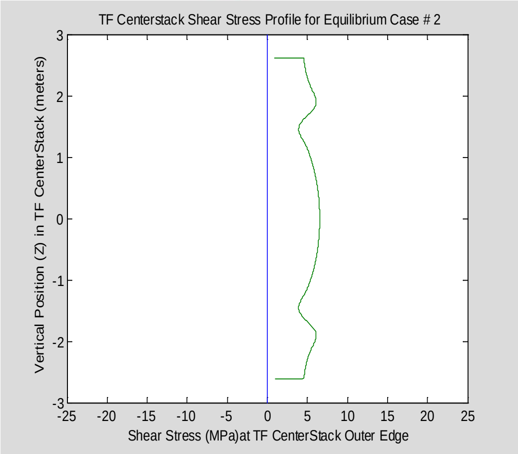
Table A3 and plots show peak shear stress is mostly due to OH, and reaches its maximum when the OH current is -24 kA. PF coils and plasma are smaller contributors to peak shear stress.

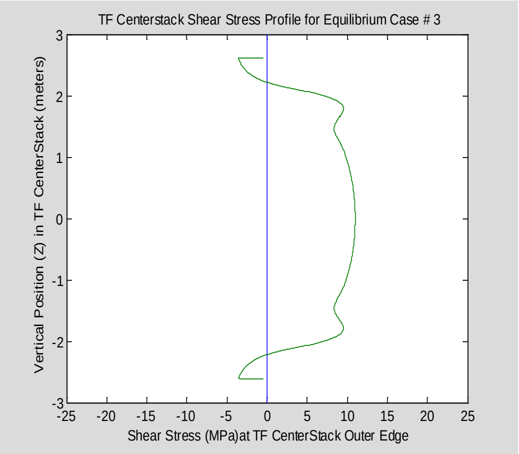
Table A3: TF Centerstack Max Absolute Shear Stress ; Precharge and Plasma Cases

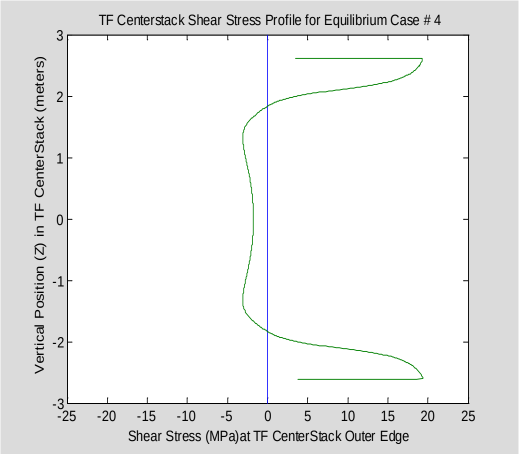
Case# -24 kA OH	Peak Shear MPa	Case# 0 kA OH	Peak Shear MPa	Case# 13 kA OH	Peak Shear MPa	Case 24 kA OH	Peak Shear MPa
						Precharge	13.160
1	19.5534	2	6.5375	3	10.9944		
4	19.4016	5	6.4449	6	10.8958		
7	19.2976	8	6.6634	9	10.9114		
10	19.1926	11	6.8980	12	11.2799		
13	19.1987	14	6.9595	15	11.3530		
16	19.5534	17	6.5375	18	10.9944		
19	18.1765	20	6.5205	21	10.9161		
22	18.5628	23	6.6753	24	11.0597		
25	18.8599	26	6.7516	27	11.1091		
28	19.0966	29	6.9223	30	11.1563		
31	19.2033	32	7.2720	33	11.3294		
34	17.6028	35	8.8235	36	13.4361		
37	17.8582	38	8.7835	39	13.3987		
40	18.1871	41	8.6822	42	13.3054		
43	18.5673	44	8.4675	45	13.1140		
46	18.8337	47	8.1257	38	12.8219		
49	16.6842	50	8.0255	51	12.6412		
52	16.9927	53	8.0063	54	12.6375		
55	17.4139	56	7.9418	57	12.5644		
58	17.8337	59	7.7767	60	12.4114		
61	18.2611	62	7.4595	63	12.1297		
64	19.5428	65	7.6528	66	12.1223		
67	19.4479	68	7.4859	69	11.9422		
70	19.0970	71	7.0231	72	11.4907		
73	18.7393	74	6.7629	75	11.2275		
76	18.4170	77	6.5354	78	10.9964		
79	17.9967	80	6.3280	81	10.7971		

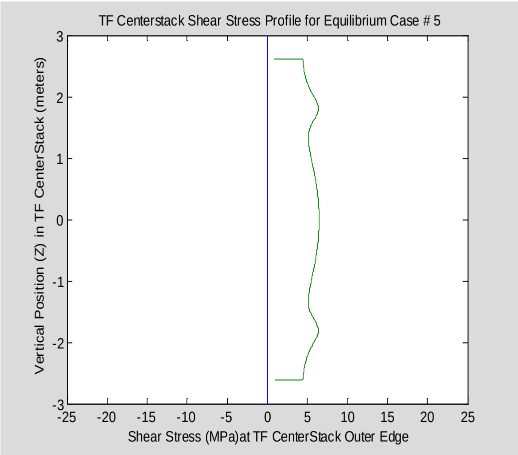
82	**19.1319**	83	**9.0096**	84	**13.3828**		
85	**19.1611**	86	**8.1527**	87	**12.5073**		
88	**19.2045**	89	**7.5824**	90	**11.9441**		
91	**19.2357**	92	**7.1888**	93	**11.5370**		
94	**19.2686**	95	**6.8490**	96	**11.1865**		

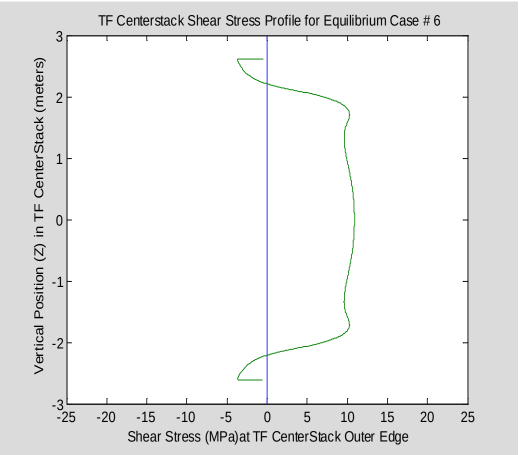

TF Centerstack Shear Stress Profile for Equilibrium Case # 1
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

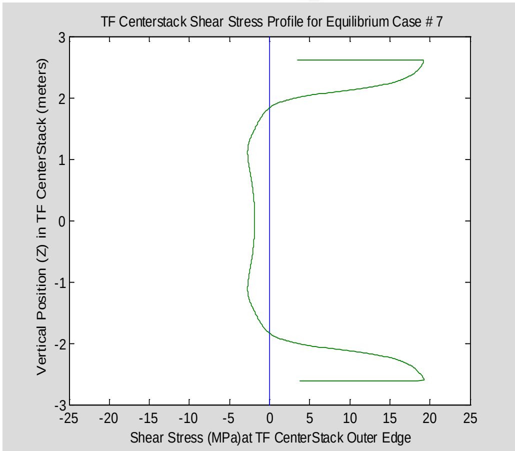

TF Centerstack Shear Stress Profile for Equilibrium Case # 2
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

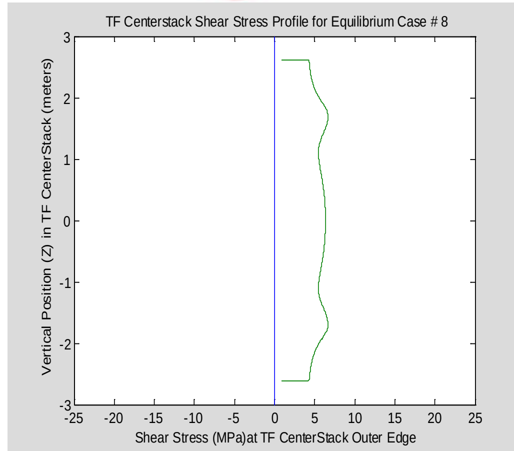

TF Centerstack Shear Stress Profile for Equilibrium Case # 3
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

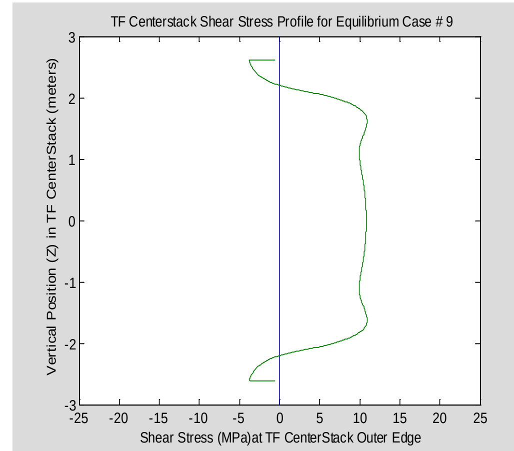

TF Centerstack Shear Stress Profile for Equilibrium Case # 4
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

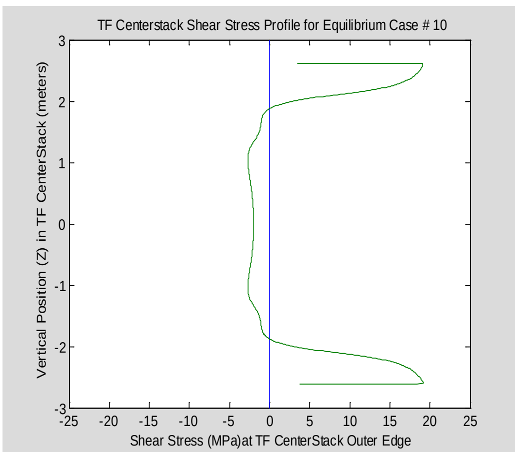

TF Centerstack Shear Stress Profile for Equilibrium Case # 5
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

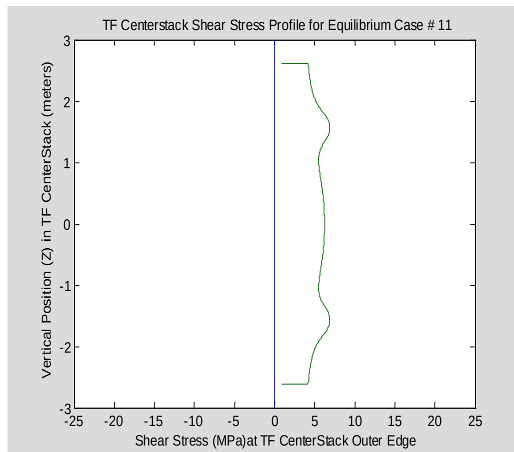

TF Centerstack Shear Stress Profile for Equilibrium Case # 6
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

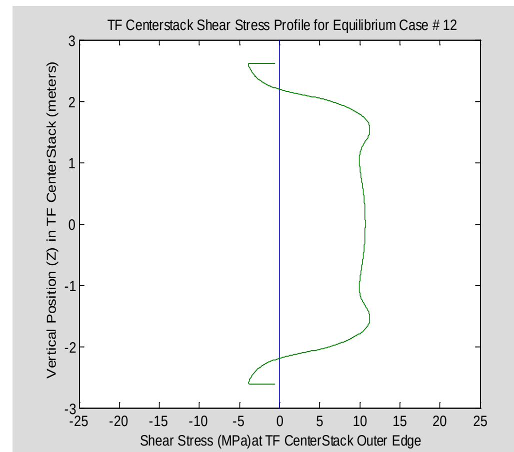

TF Centerstack Shear Stress Profile for Equilibrium Case # 7
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

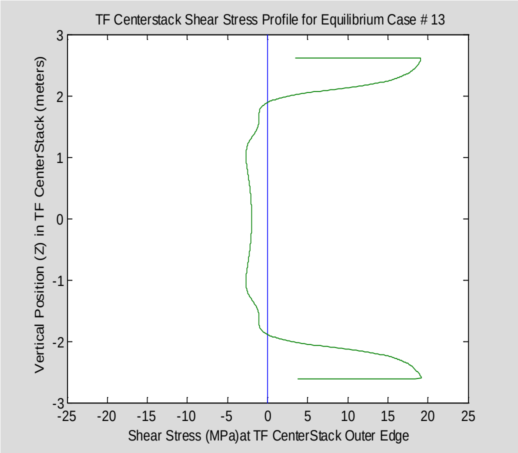

TF Centerstack Shear Stress Profile for Equilibrium Case # 8
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

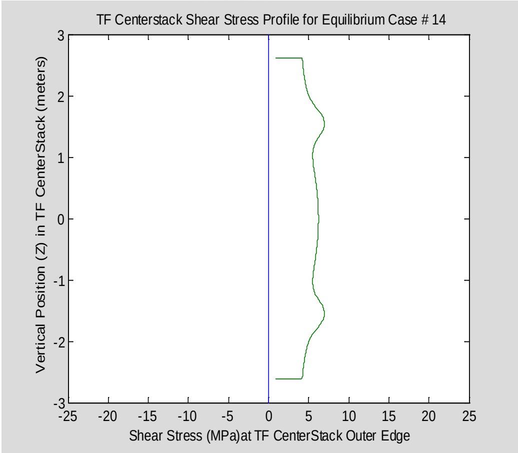

TF Centerstack Shear Stress Profile for Equilibrium Case # 9
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

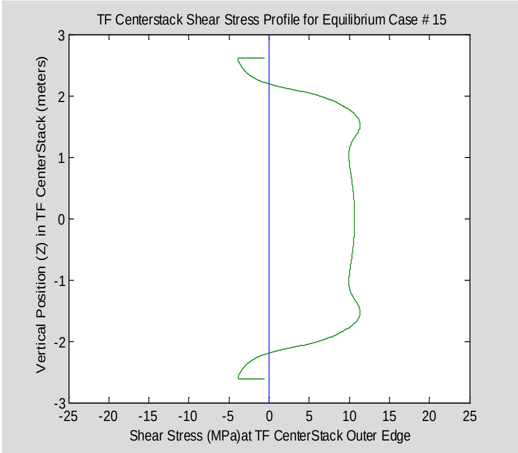

TF Centerstack Shear Stress Profile for Equilibrium Case # 10
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

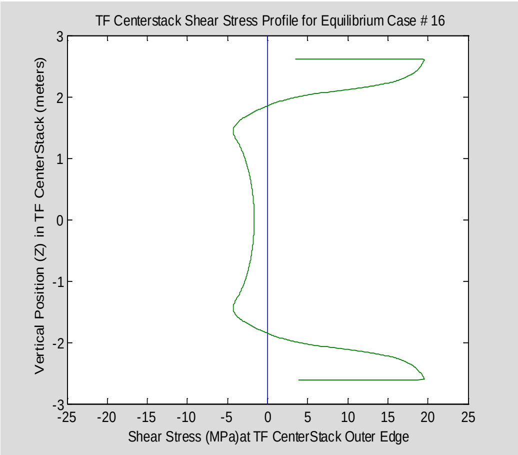

TF Centerstack Shear Stress Profile for Equilibrium Case # 11
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

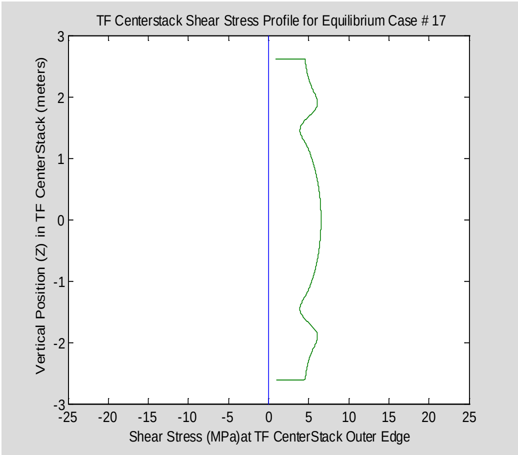

TF Centerstack Shear Stress Profile for Equilibrium Case # 12
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

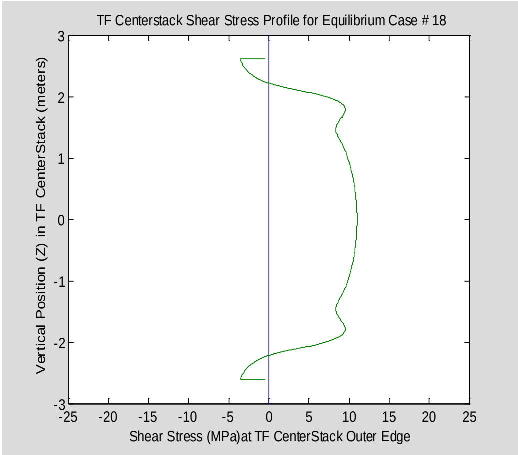

TF Centerstack Shear Stress Profile for Equilibrium Case # 13
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-25 -20 -15 -10 -5 0 5 10 15 20 25

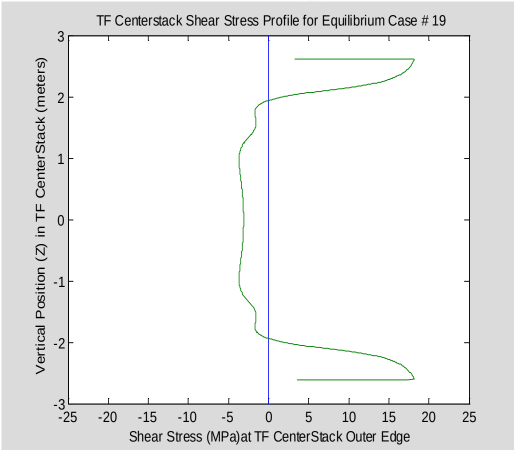

TF Centerstack Shear Stress Profile for Equilibrium Case # 14
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-25 -20 -15 -10 -5 0 5 10 15 20 25

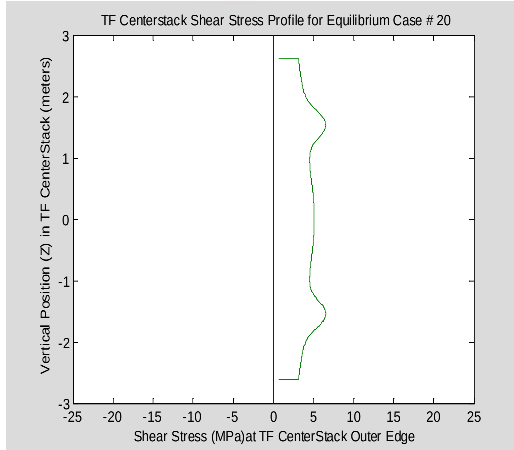

TF Centerstack Shear Stress Profile for Equilibrium Case # 15
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-25 -20 -15 -10 -5 0 5 10 15 20 25

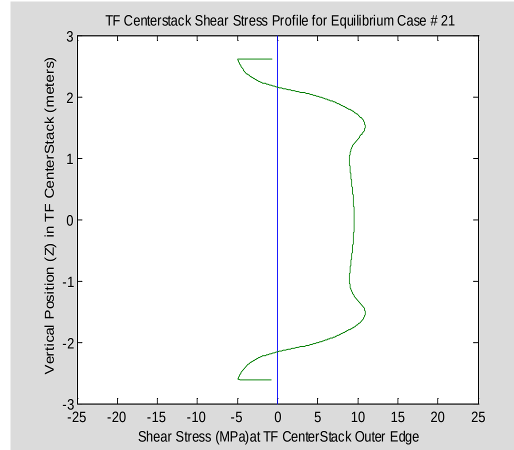

TF Centerstack Shear Stress Profile for Equilibrium Case # 16
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-25 -20 -15 -10 -5 0 5 10 15 20 25

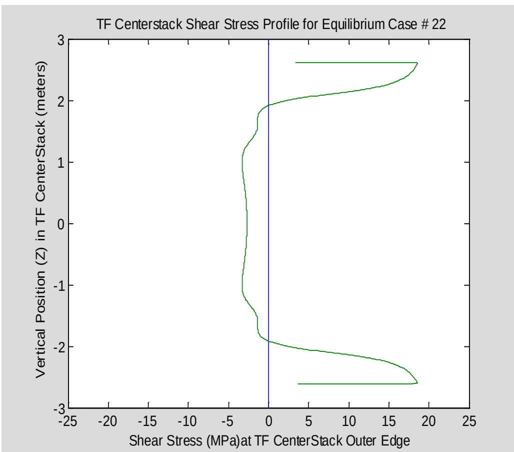

TF Centerstack Shear Stress Profile for Equilibrium Case # 17
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-25 -20 -15 -10 -5 0 5 10 15 20 25

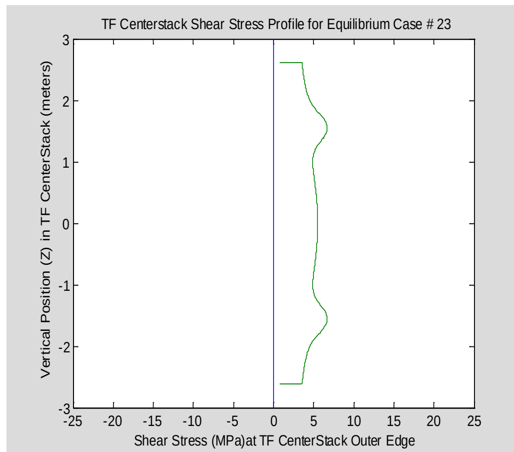

TF Centerstack Shear Stress Profile for Equilibrium Case # 18
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-25 -20 -15 -10 -5 0 5 10 15 20 25

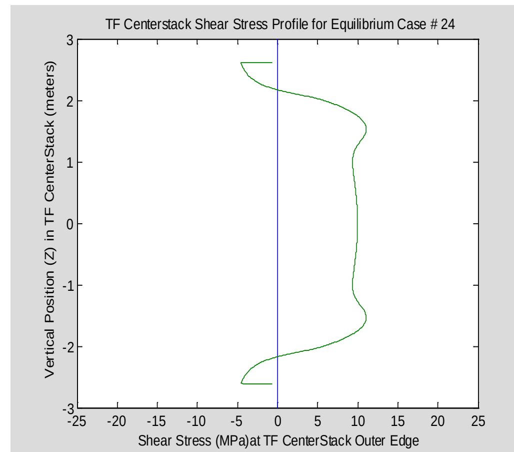

TF Centerstack Shear Stress Profile for Equilibrium Case # 19
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

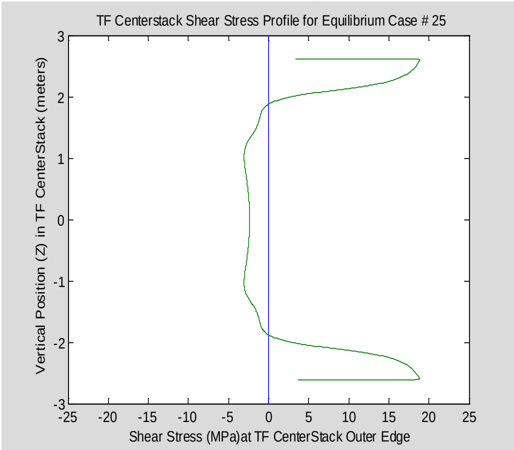

TF Centerstack Shear Stress Profile for Equilibrium Case # 20
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

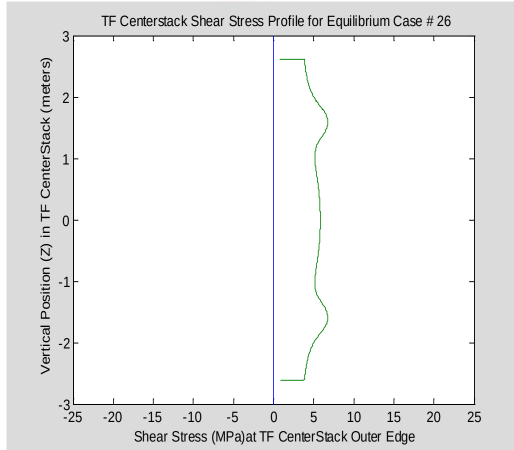

TF Centerstack Shear Stress Profile for Equilibrium Case # 21
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

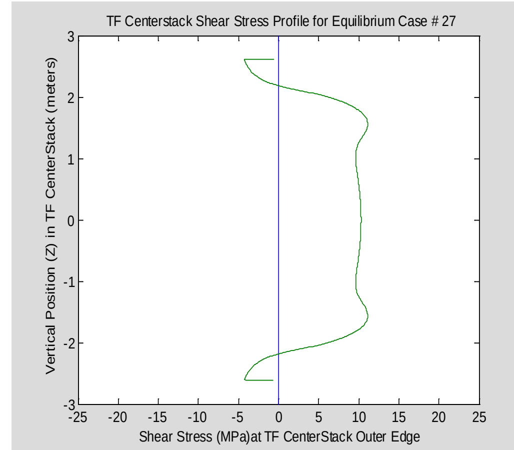

TF Centerstack Shear Stress Profile for Equilibrium Case # 22
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

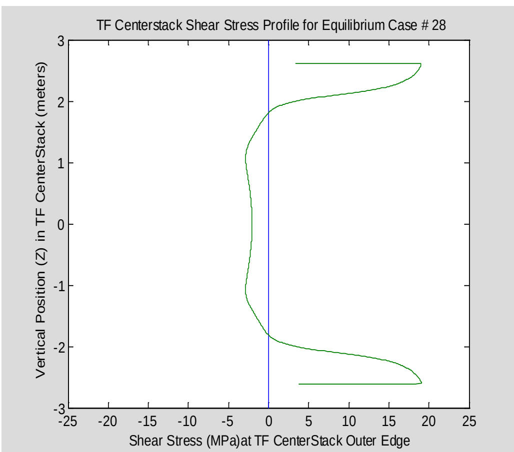

TF Centerstack Shear Stress Profile for Equilibrium Case # 23
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

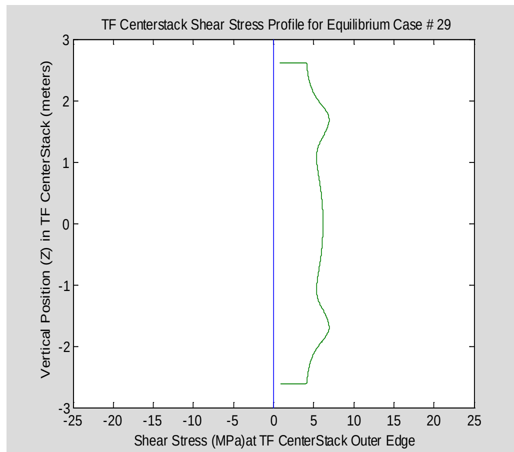

TF Centerstack Shear Stress Profile for Equilibrium Case # 24
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

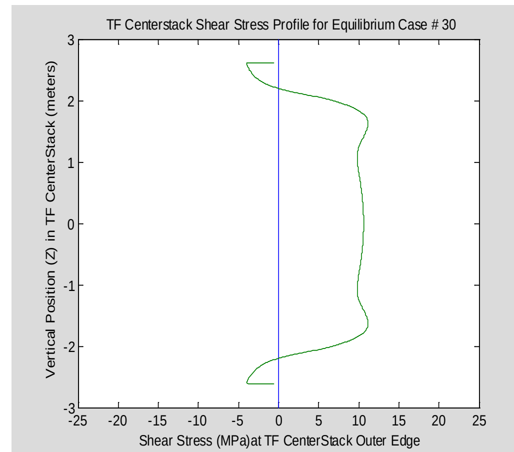

TF Centerstack Shear Stress Profile for Equilibrium Case # 25
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

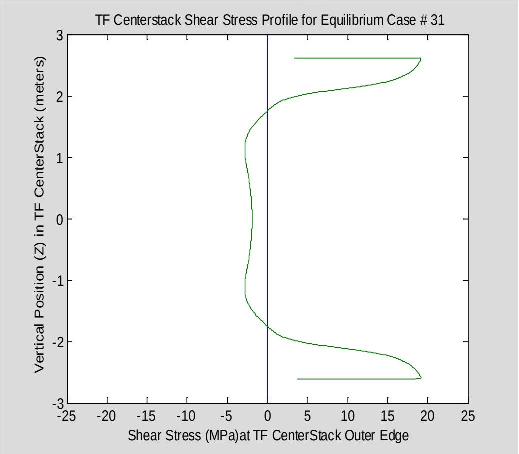

TF Centerstack Shear Stress Profile for Equilibrium Case # 26
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

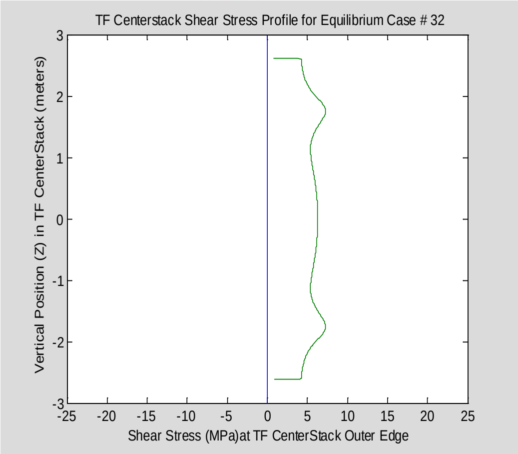

TF Centerstack Shear Stress Profile for Equilibrium Case # 27
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

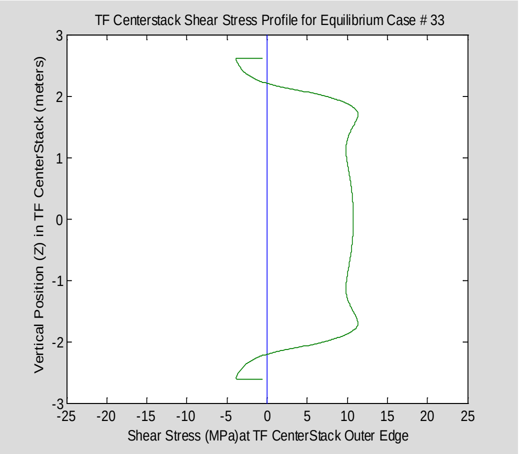

TF Centerstack Shear Stress Profile for Equilibrium Case # 28
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

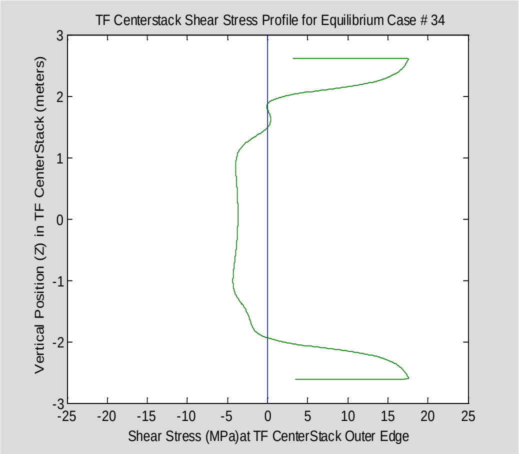

TF Centerstack Shear Stress Profile for Equilibrium Case # 29
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

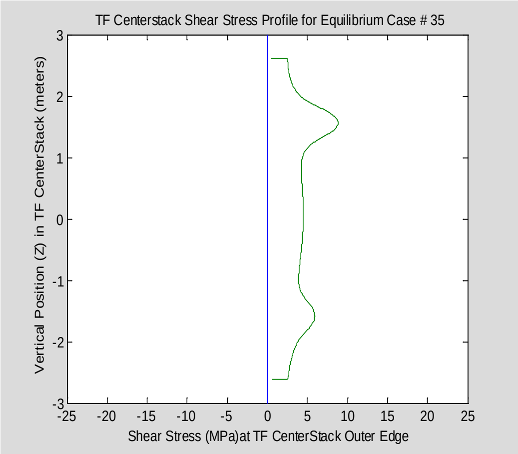

TF Centerstack Shear Stress Profile for Equilibrium Case # 30
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

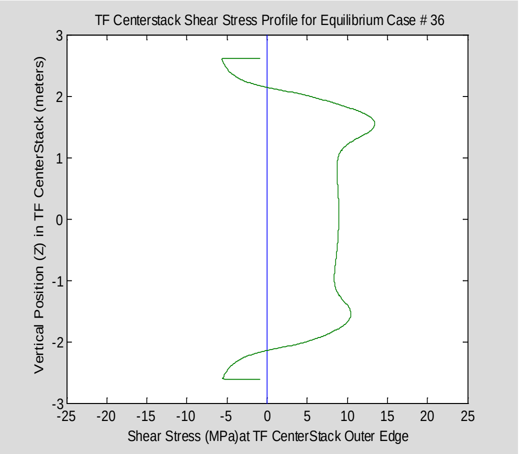

TF Centerstack Shear Stress Profile for Equilibrium Case # 31
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

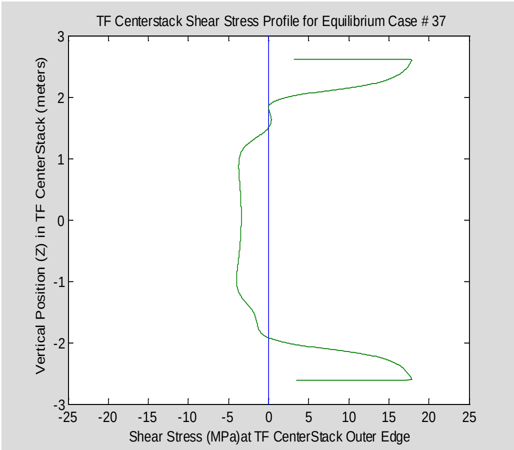

TF Centerstack Shear Stress Profile for Equilibrium Case # 32
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

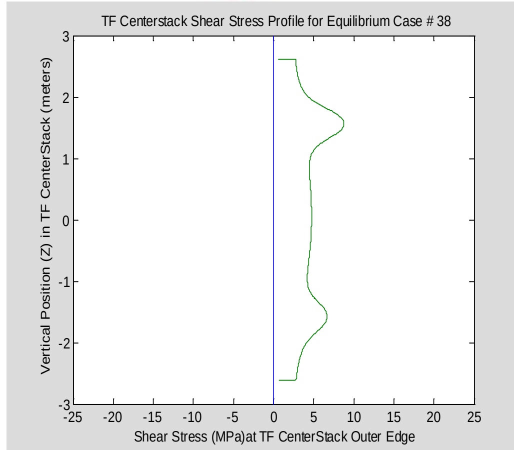

TF Centerstack Shear Stress Profile for Equilibrium Case # 33
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

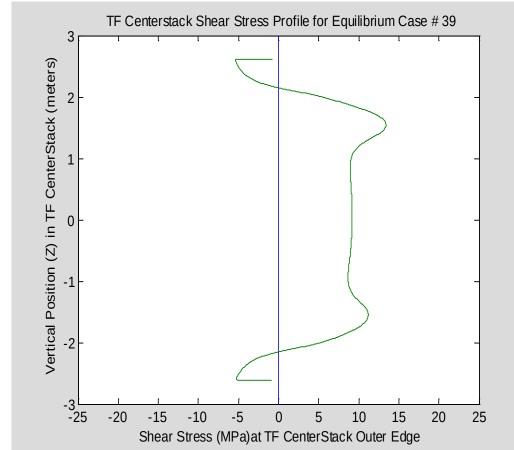

TF Centerstack Shear Stress Profile for Equilibrium Case # 34
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

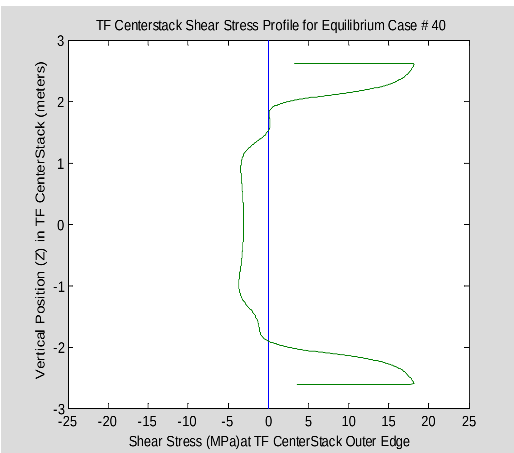

TF Centerstack Shear Stress Profile for Equilibrium Case # 35
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

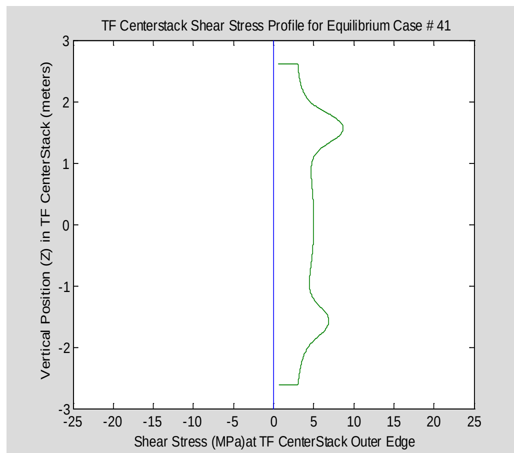

TF Centerstack Shear Stress Profile for Equilibrium Case # 36
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

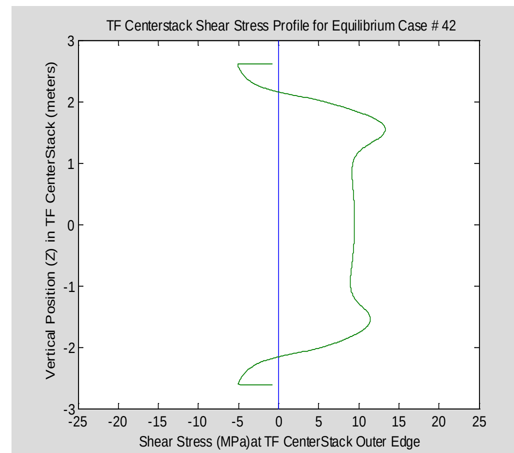

TF Centerstack Shear Stress Profile for Equilibrium Case # 37
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

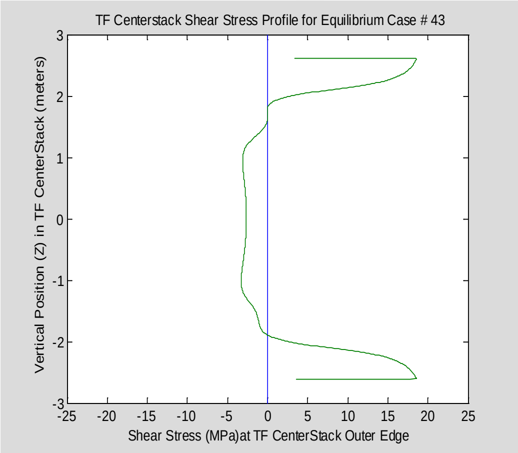

TF Centerstack Shear Stress Profile for Equilibrium Case # 38
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

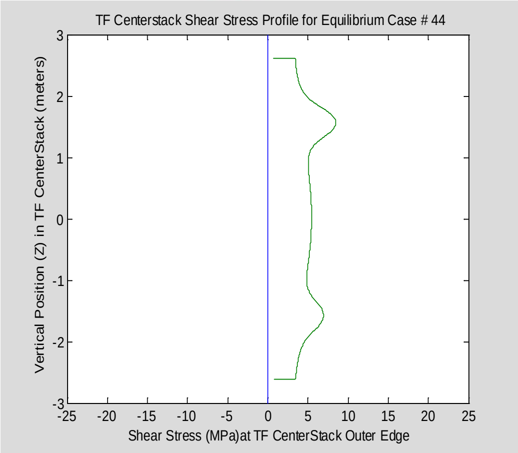

TF Centerstack Shear Stress Profile for Equilibrium Case # 39
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

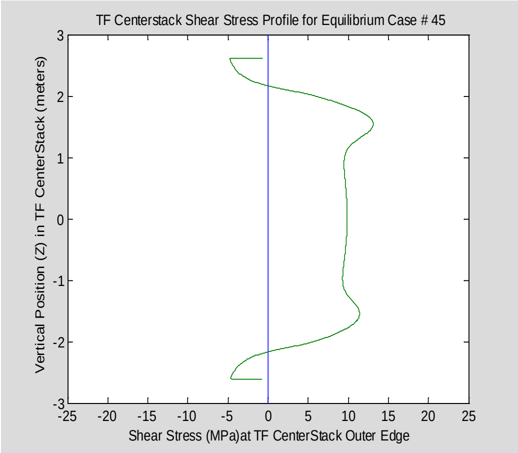

TF Centerstack Shear Stress Profile for Equilibrium Case # 40
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

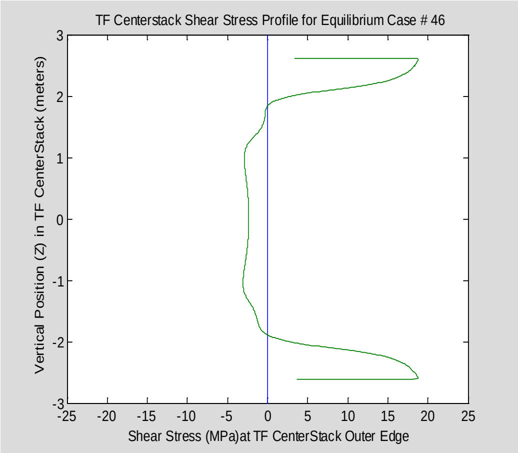

TF Centerstack Shear Stress Profile for Equilibrium Case # 41
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

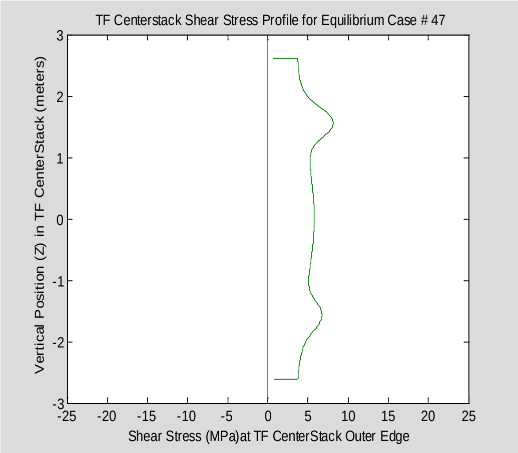

TF Centerstack Shear Stress Profile for Equilibrium Case # 42
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

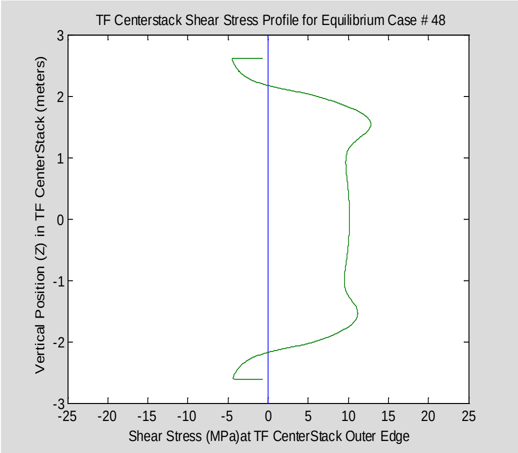

TF Centerstack Shear Stress Profile for Equilibrium Case # 43
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

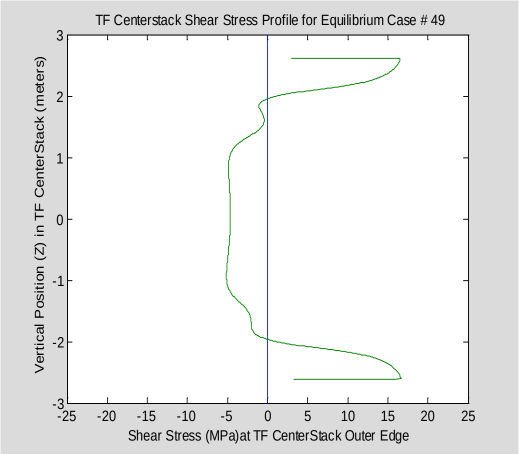

TF Centerstack Shear Stress Profile for Equilibrium Case # 44
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

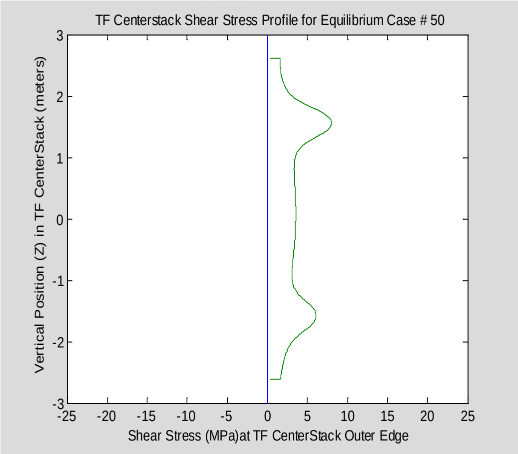

TF Centerstack Shear Stress Profile for Equilibrium Case # 45
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

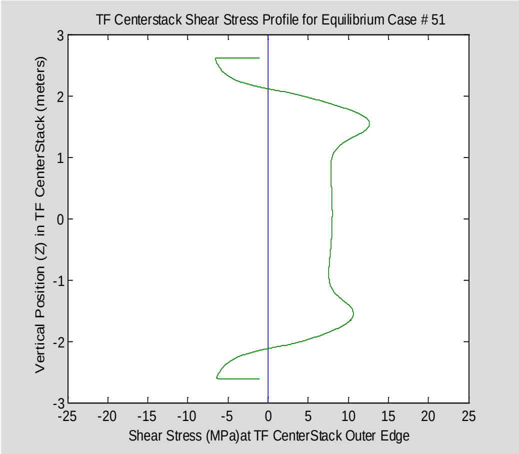

TF Centerstack Shear Stress Profile for Equilibrium Case # 46
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

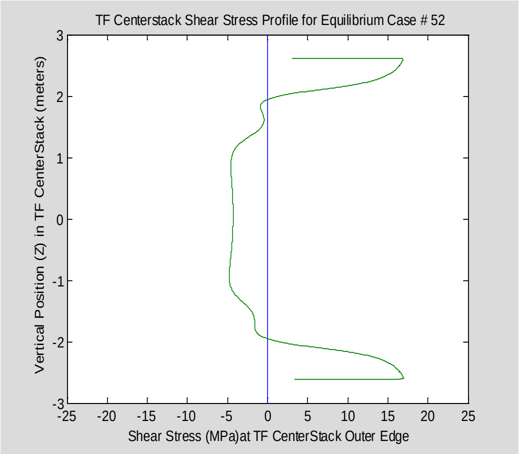

TF Centerstack Shear Stress Profile for Equilibrium Case # 47
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

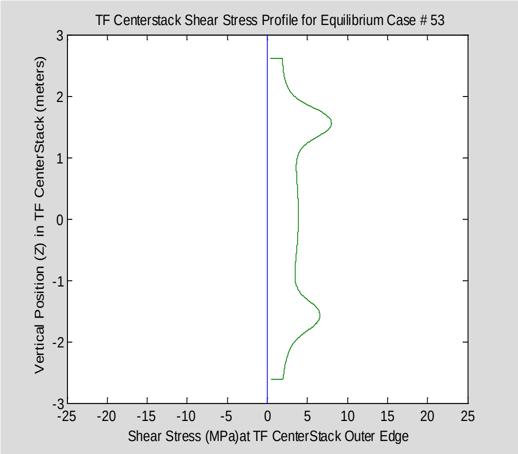

TF Centerstack Shear Stress Profile for Equilibrium Case # 48
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

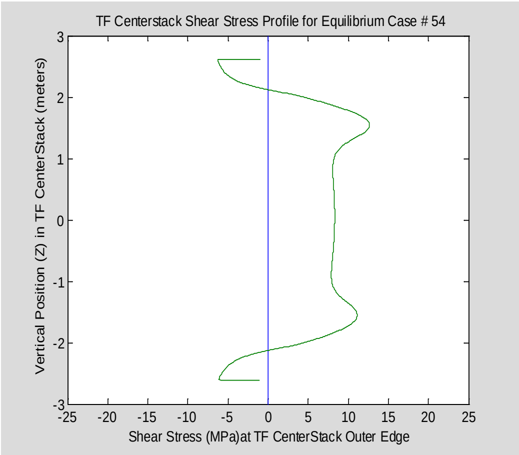

TF Centerstack Shear Stress Profile for Equilibrium Case # 49
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

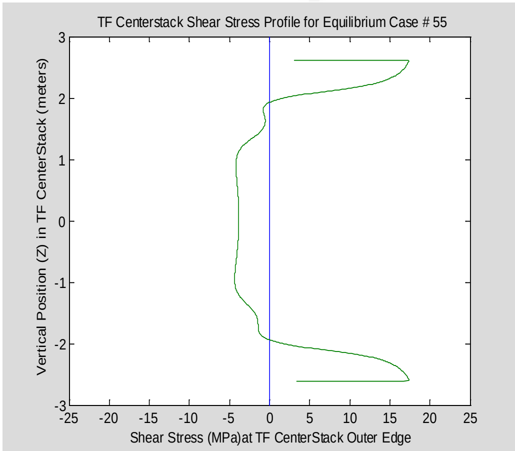

TF Centerstack Shear Stress Profile for Equilibrium Case # 50
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

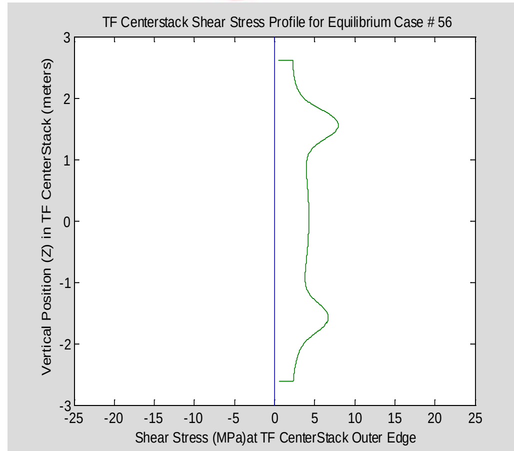

TF Centerstack Shear Stress Profile for Equilibrium Case # 51
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

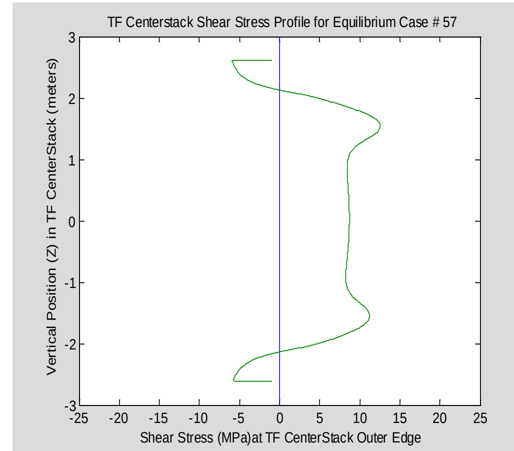

TF Centerstack Shear Stress Profile for Equilibrium Case # 52
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

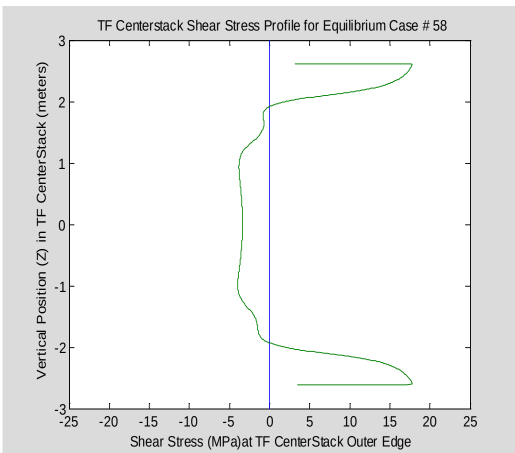

TF Centerstack Shear Stress Profile for Equilibrium Case # 53
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

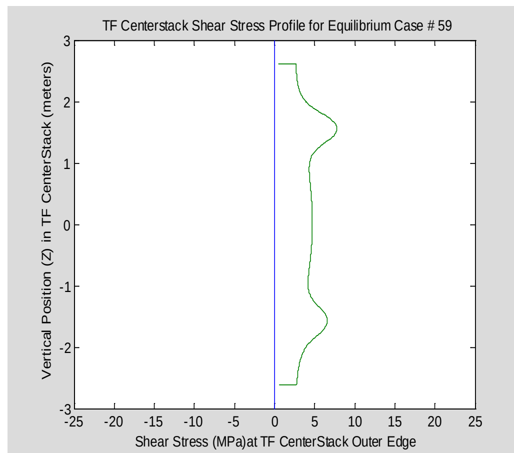

TF Centerstack Shear Stress Profile for Equilibrium Case # 54
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

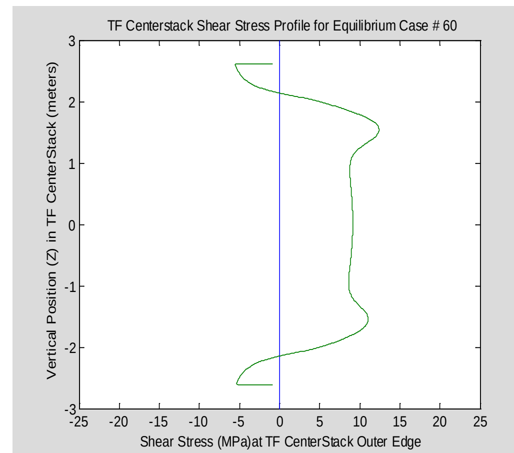

TF Centerstack Shear Stress Profile for Equilibrium Case # 55
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

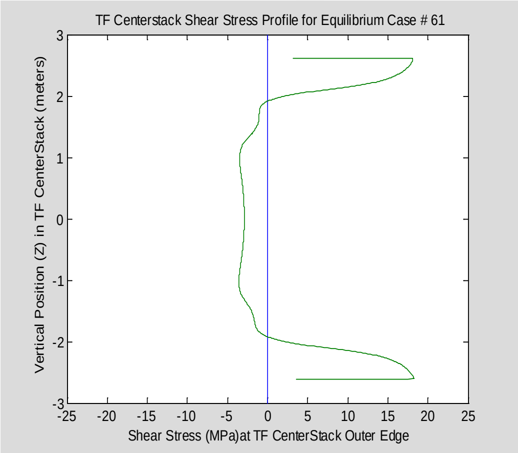

TF Centerstack Shear Stress Profile for Equilibrium Case # 56
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

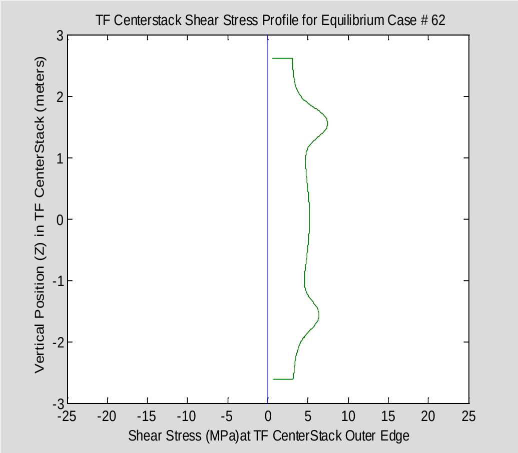

TF Centerstack Shear Stress Profile for Equilibrium Case # 57
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

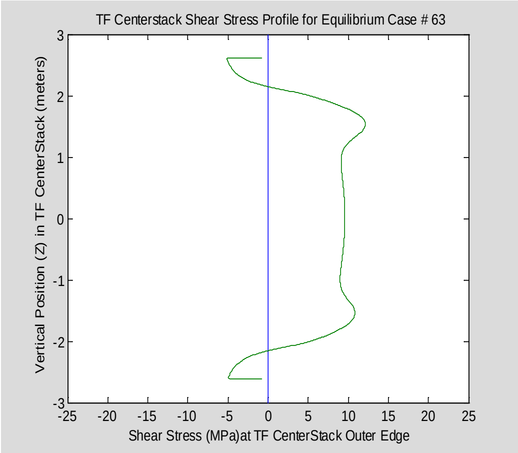

TF Centerstack Shear Stress Profile for Equilibrium Case # 58
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

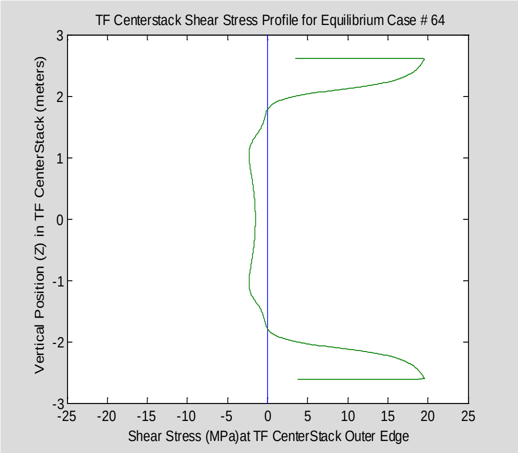

TF Centerstack Shear Stress Profile for Equilibrium Case # 59
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

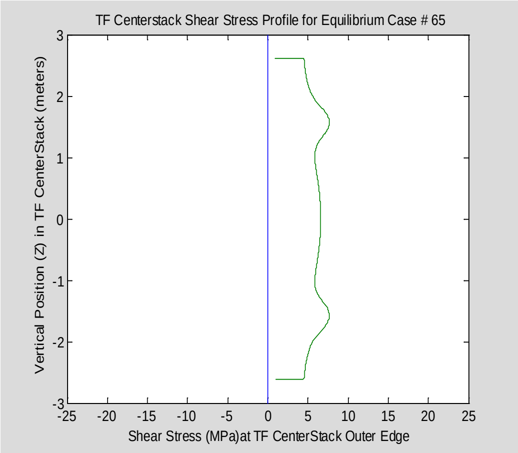

TF Centerstack Shear Stress Profile for Equilibrium Case # 60
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

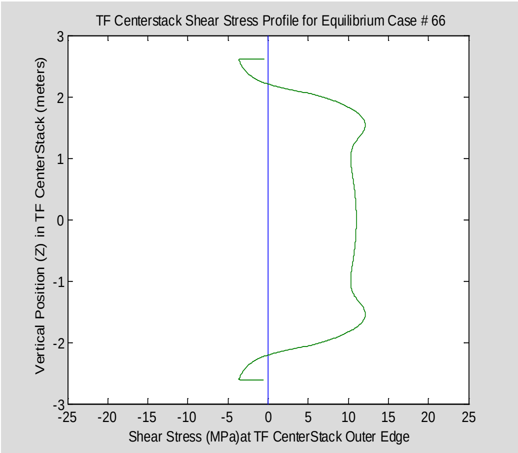

TF Centerstack Shear Stress Profile for Equilibrium Case # 61
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

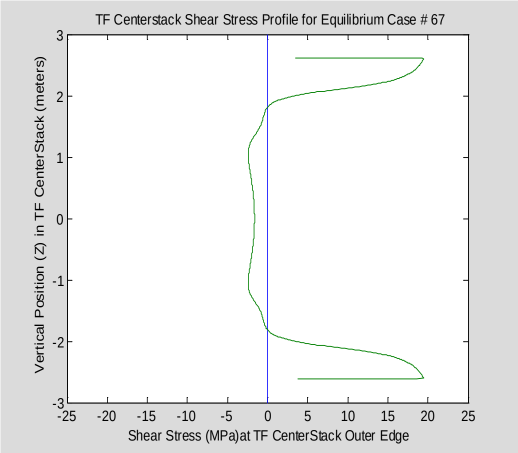

TF Centerstack Shear Stress Profile for Equilibrium Case # 62
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

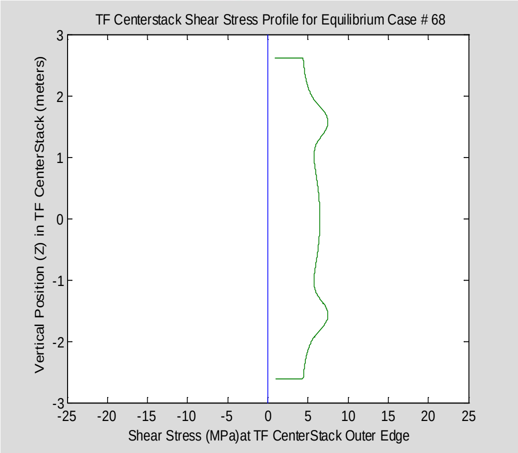

TF Centerstack Shear Stress Profile for Equilibrium Case # 63
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

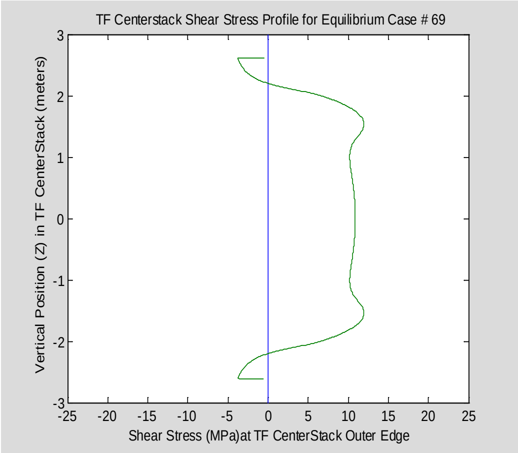

TF Centerstack Shear Stress Profile for Equilibrium Case # 64
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

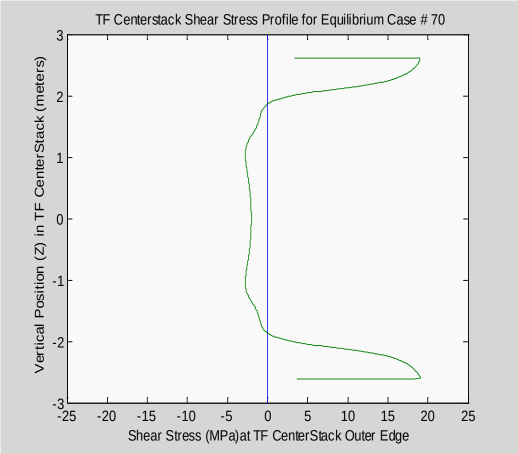

TF Centerstack Shear Stress Profile for Equilibrium Case # 65
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

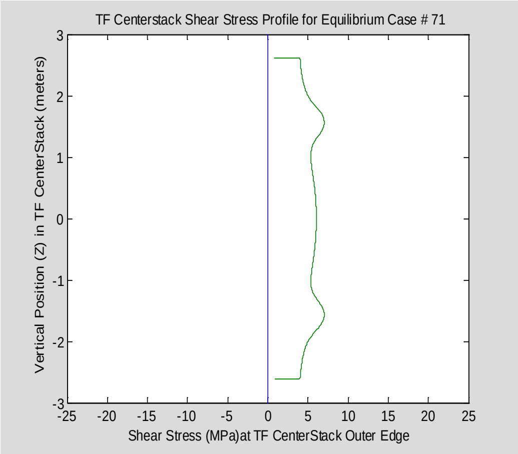

TF Centerstack Shear Stress Profile for Equilibrium Case # 66
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

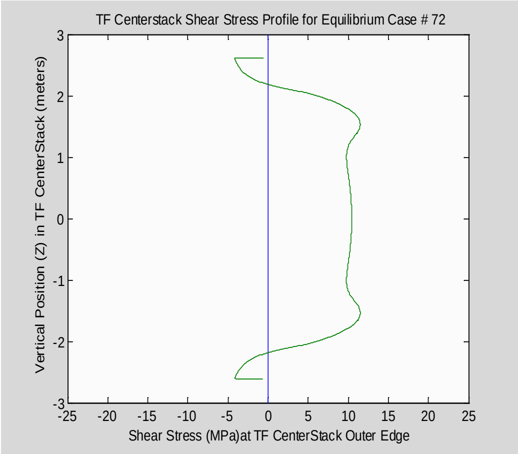

TF Centerstack Shear Stress Profile for Equilibrium Case # 67
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

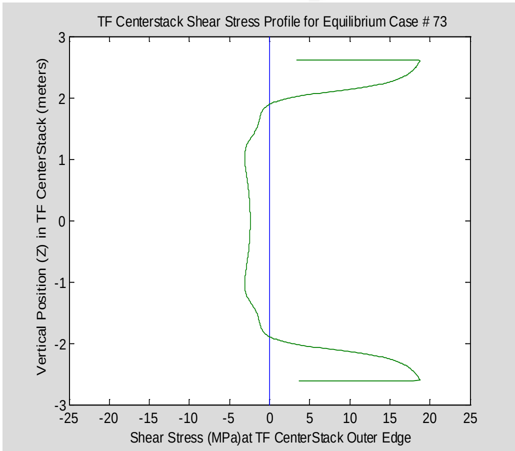

TF Centerstack Shear Stress Profile for Equilibrium Case # 68
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

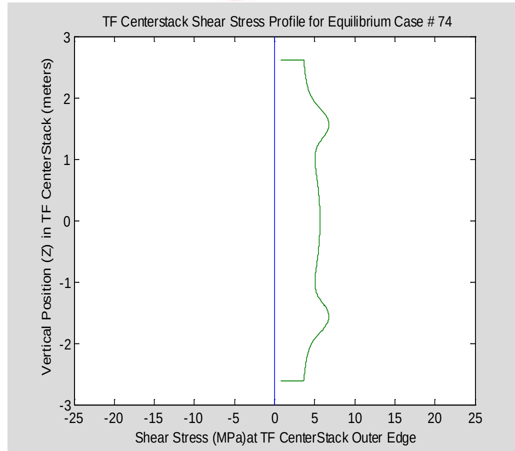

TF Centerstack Shear Stress Profile for Equilibrium Case # 69
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

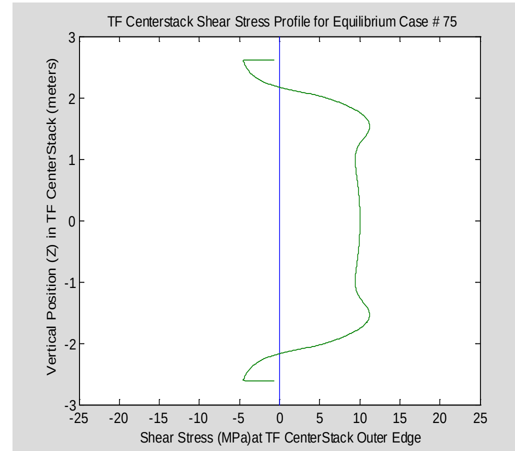

TF Centerstack Shear Stress Profile for Equilibrium Case # 70
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

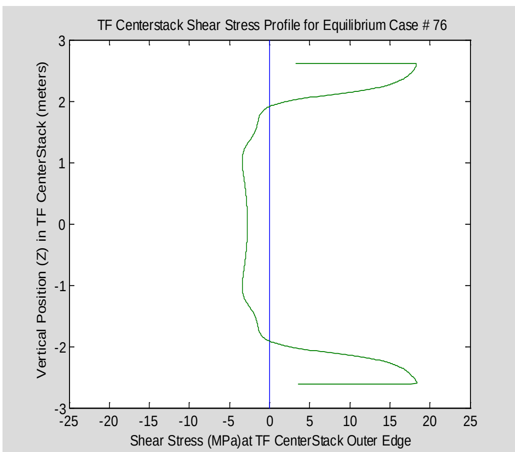

TF Centerstack Shear Stress Profile for Equilibrium Case # 71
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

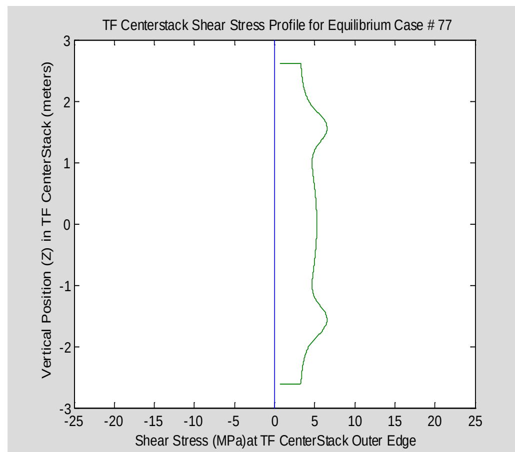

TF Centerstack Shear Stress Profile for Equilibrium Case # 72
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

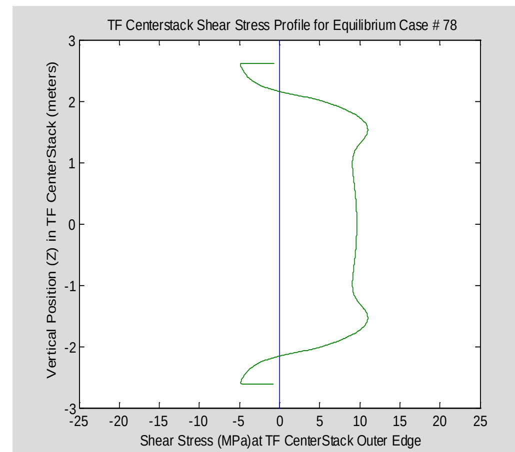

TF Centerstack Shear Stress Profile for Equilibrium Case # 73
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

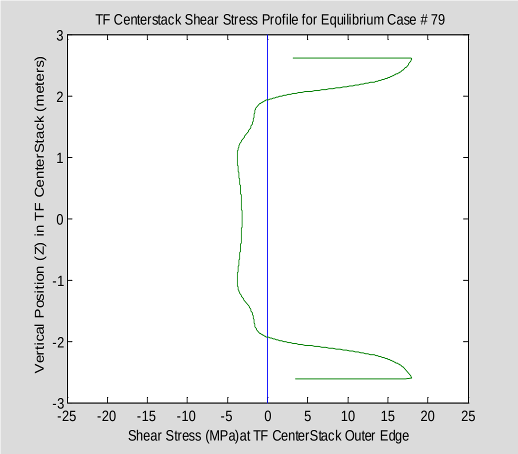

TF Centerstack Shear Stress Profile for Equilibrium Case # 74
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

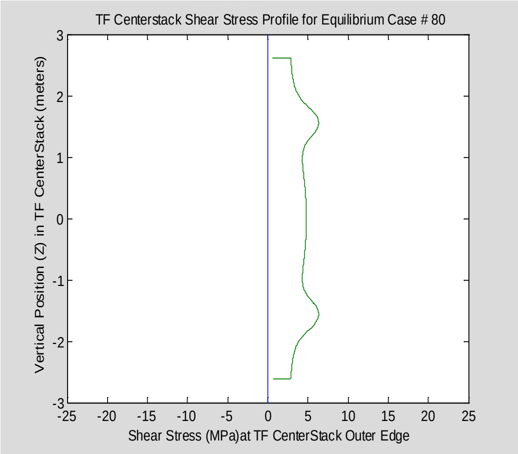

TF Centerstack Shear Stress Profile for Equilibrium Case # 75
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

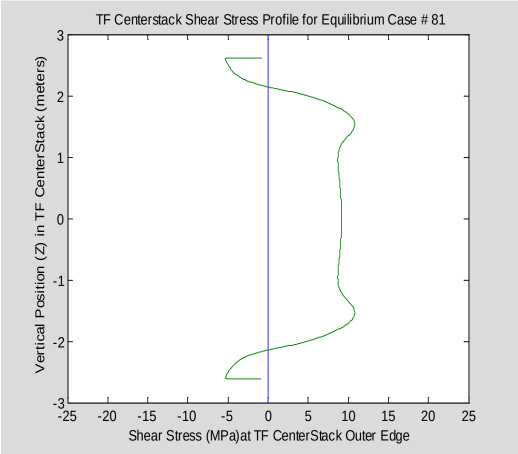

TF Centerstack Shear Stress Profile for Equilibrium Case # 76
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

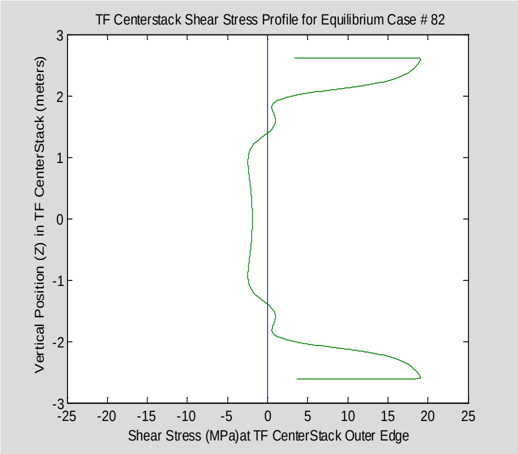

TF Centerstack Shear Stress Profile for Equilibrium Case # 77
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

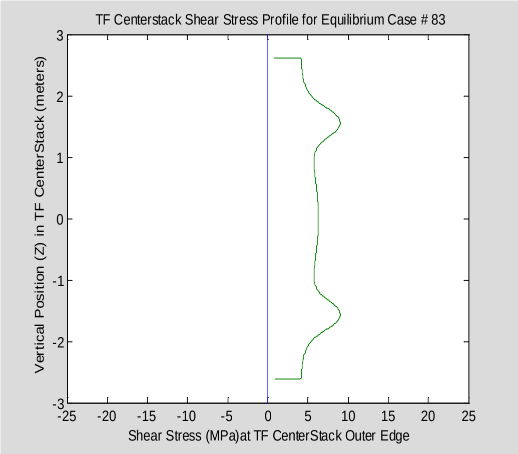

TF Centerstack Shear Stress Profile for Equilibrium Case # 78
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

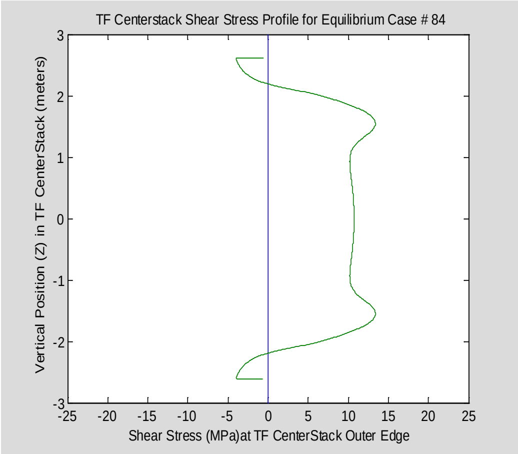

TF Centerstack Shear Stress Profile for Equilibrium Case # 79
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

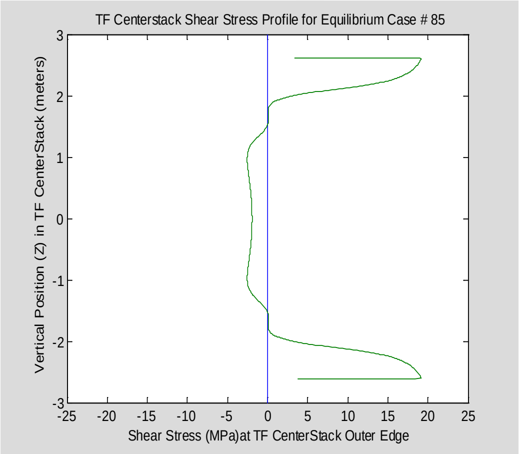

TF Centerstack Shear Stress Profile for Equilibrium Case # 80
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 81
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

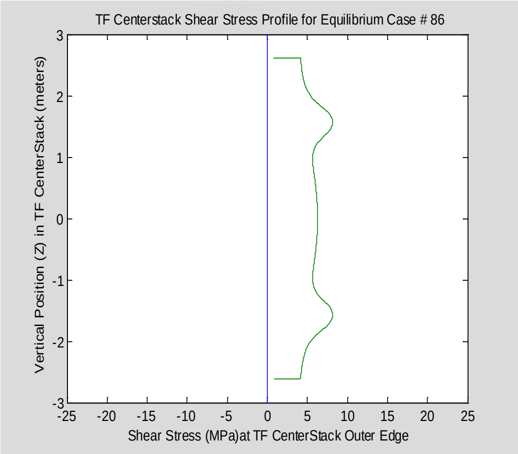

TF Centerstack Shear Stress Profile for Equilibrium Case # 82
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 83
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

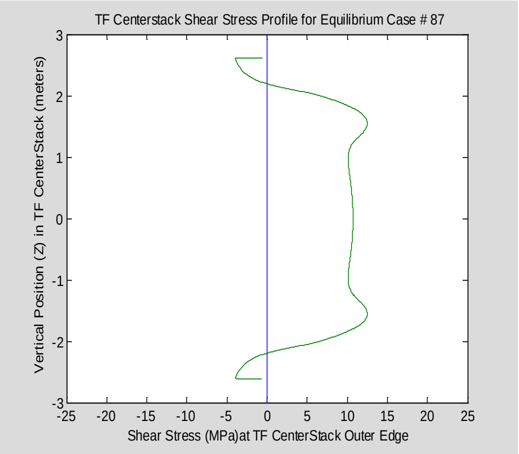

TF Centerstack Shear Stress Profile for Equilibrium Case # 84
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge



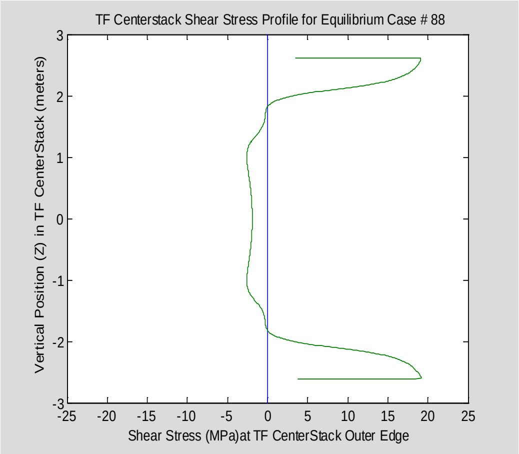
TF Centerstack Shear Stress Profile for Equilibrium Case # 85
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge



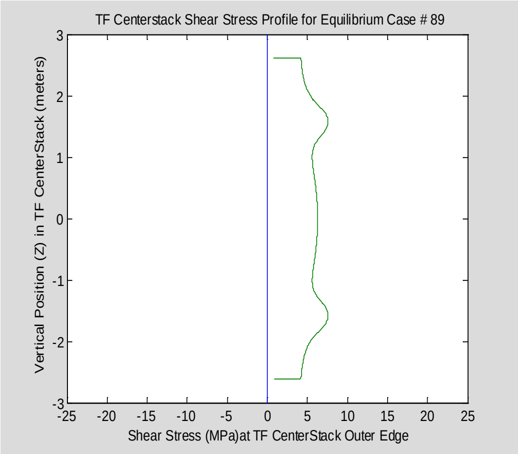
TF Centerstack Shear Stress Profile for Equilibrium Case # 86
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge



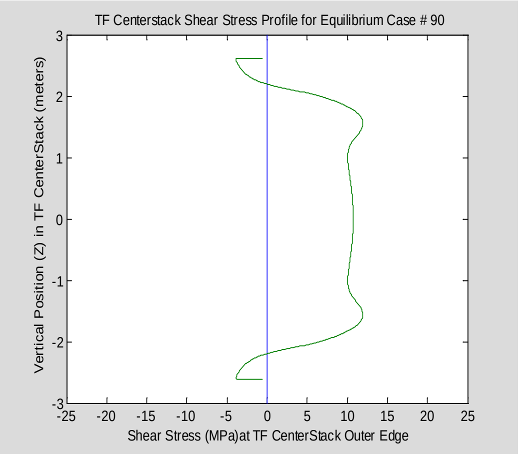
TF Centerstack Shear Stress Profile for Equilibrium Case # 87
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge



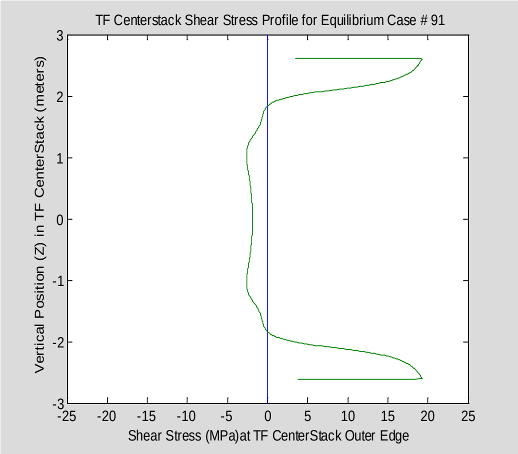
TF Centerstack Shear Stress Profile for Equilibrium Case # 88
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

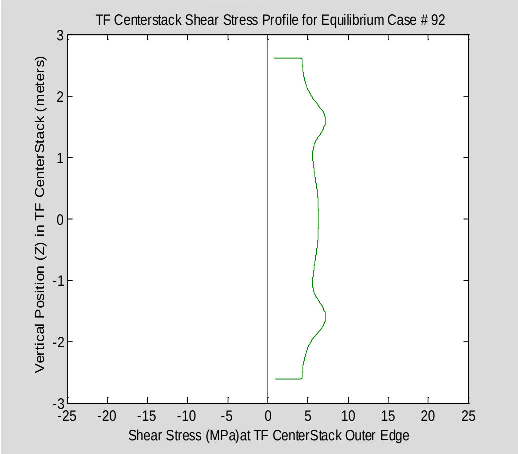


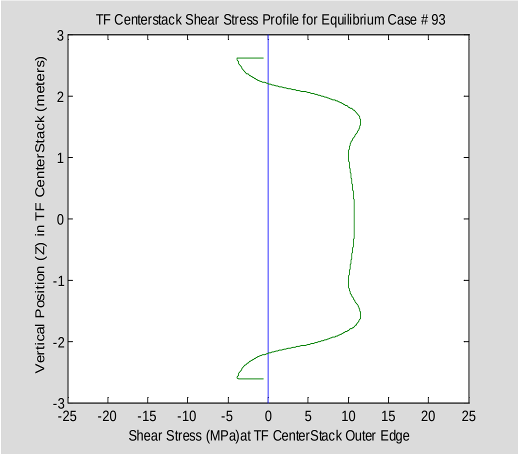
TF Centerstack Shear Stress Profile for Equilibrium Case # 89
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

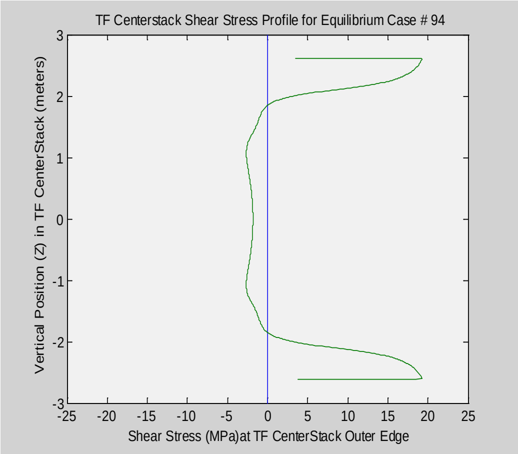


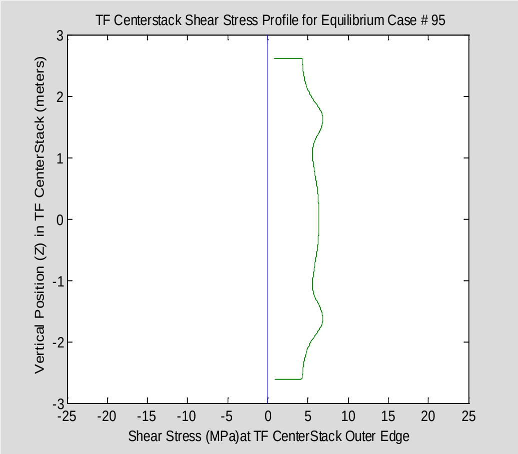
TF Centerstack Shear Stress Profile for Equilibrium Case # 90
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

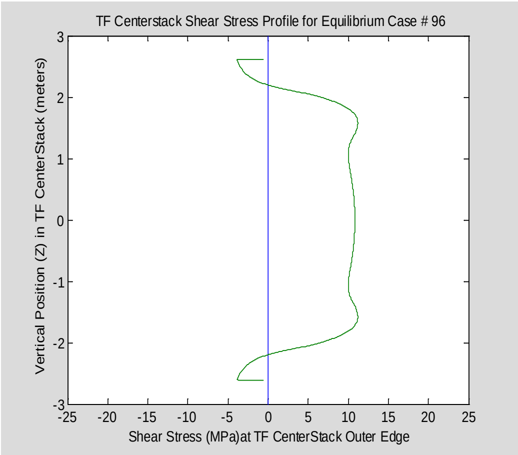

TF Centerstack Shear Stress Profile for Equilibrium Case # 91
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge

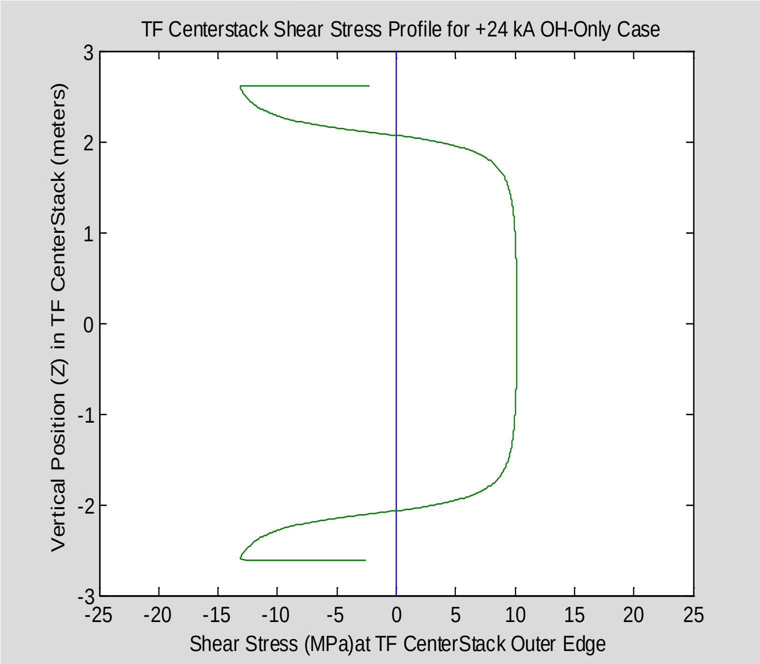

TF Centerstack Shear Stress Profile for Equilibrium Case # 92
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 93
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 94
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 95
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for Equilibrium Case # 96
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge


TF Centerstack Shear Stress Profile for +24 kA OH-Only Case
Vertical Position (Z) in TF CenterStack (meters)
Shear Stress (MPa)at TF CenterStack Outer Edge
3
2
1
0
-1
-2
-3
-25
-20
-15
-10
-5
0
5
10
15
20
25